An introduction to structural equation modeling

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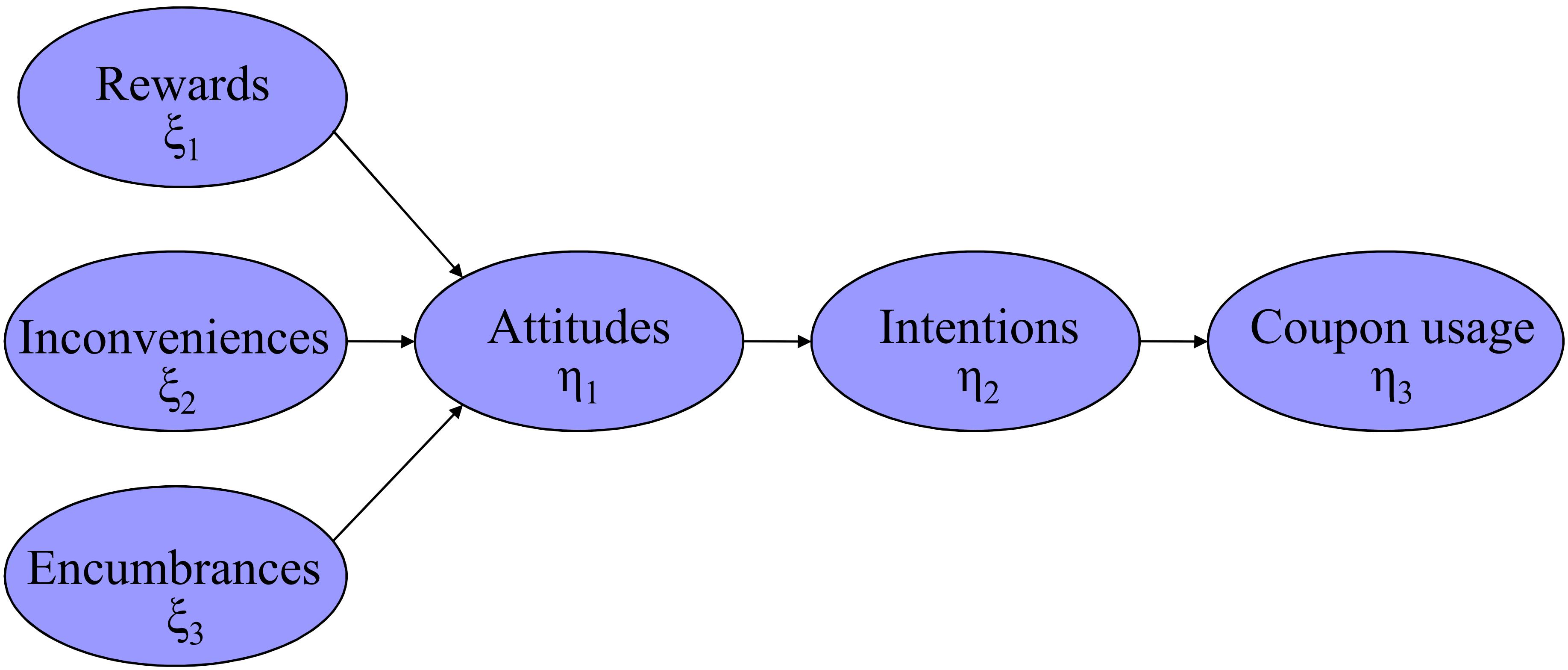
Structural equation modeling (SEM)

- also known as latent variable modeling, latent variable path analysis, (means and) covariance (or moment) structure analysis, causal modeling, etc.;
- a technique for investigating relationships between latent (unobserved) variables or constructs that are measured by (multiple) manifest (observed) variables or indicators;
- can be thought of as a combination of regression analysis (including systems of simultaneous equations) and factor analysis;
- special cases are confirmatory factor analysis and manifest variable path analysis;
- in recent years, SEM has been extended in many ways;

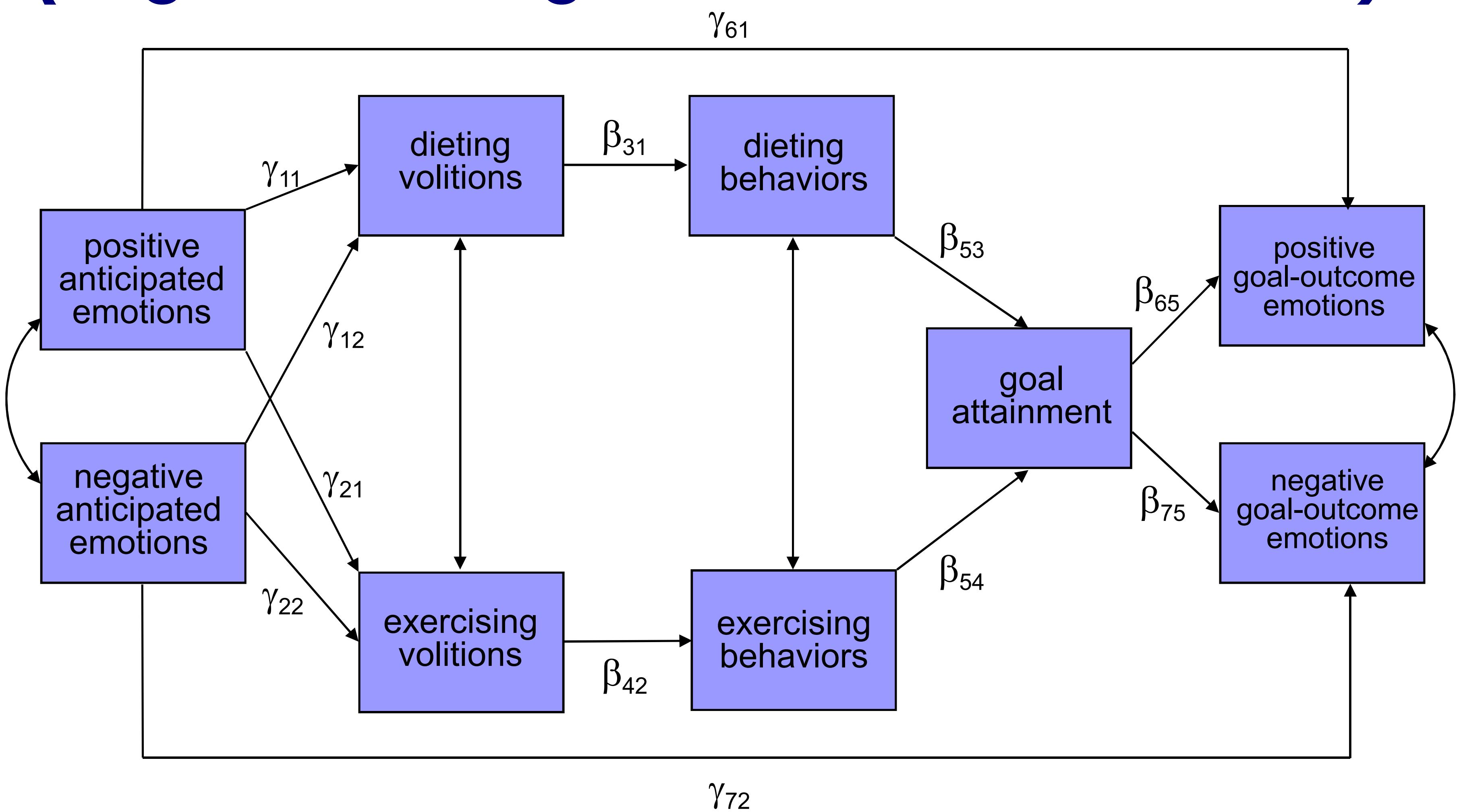
SEM (cont'd)

- two primary advantages of SEM:
  - SEM makes it possible to study complex patterns of relationships among the constructs in a conceptual model in an integrative fashion;
  - the measurement of unobserved (latent) variables by observed fallible indicators can be modeled explicitly, and the effect of measurement error (both random and systematic) on structural relationships can be taken into account;

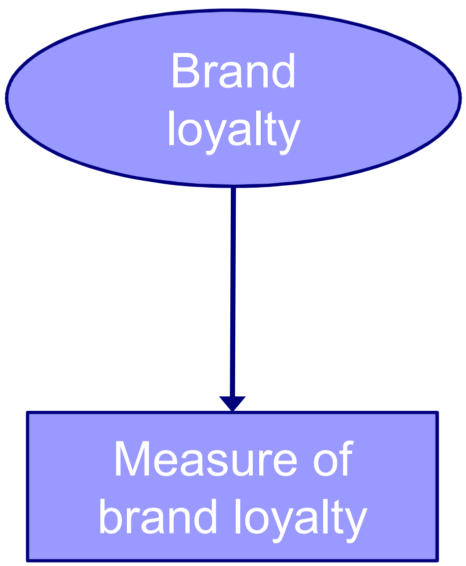
Explaining the usage of coupons for grocery shopping (cf. Bagozzi, Baumgartner, and Yi 1992)



Goal-directed emotions (Bagozzi, Baumgartner, and Pieters 1998)

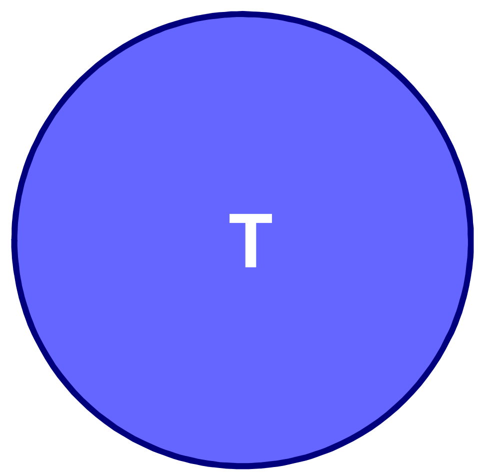


The relationship between observed measurements and constructs of interest

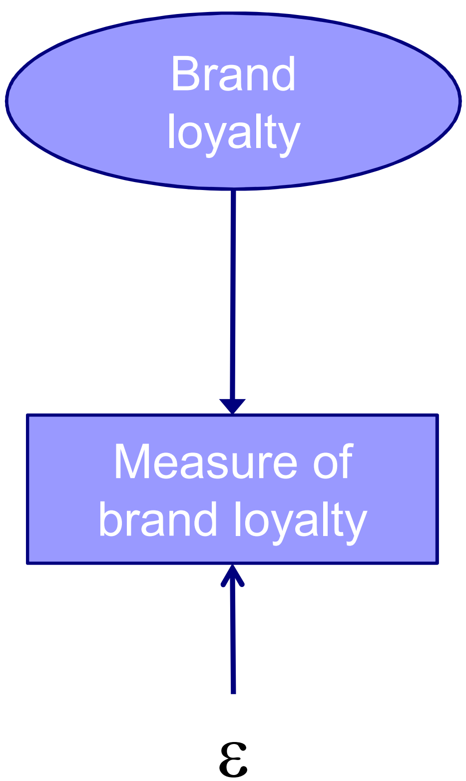


(e.g., I think of myself as a brand-loyal consumer.)

- The observed single-item brand loyalty score is a perfect measure of “true” brand loyalty.
- All of the variability in observed scores is trait (substantive) variance.

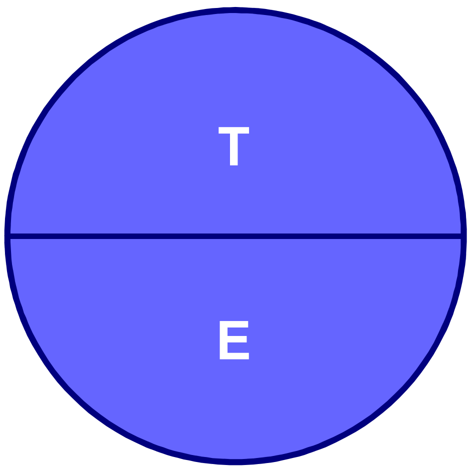


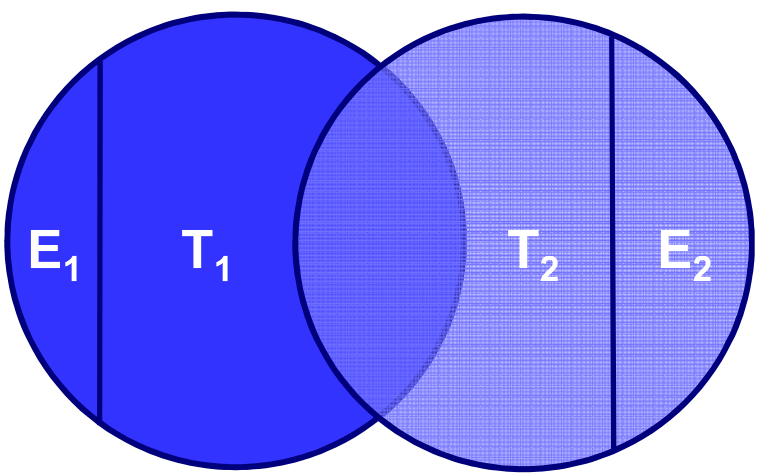
The relationship between observed measurements and constructs of interest (cont'd)



- The observed brand loyalty score is contaminated by random measurement error.
- If only a single measure is available, random measurement error cannot be taken into account.

The relationship between observed measurements and constructs of interest (cont'd)

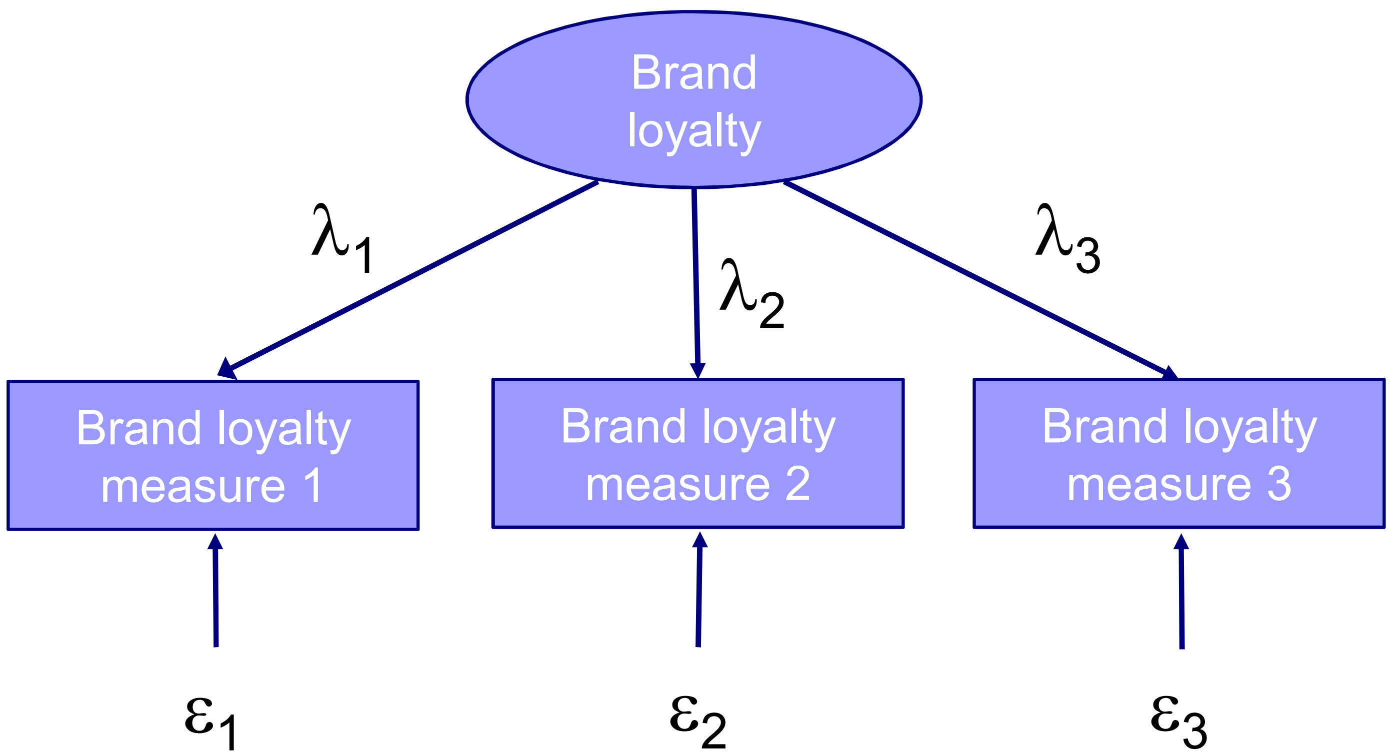




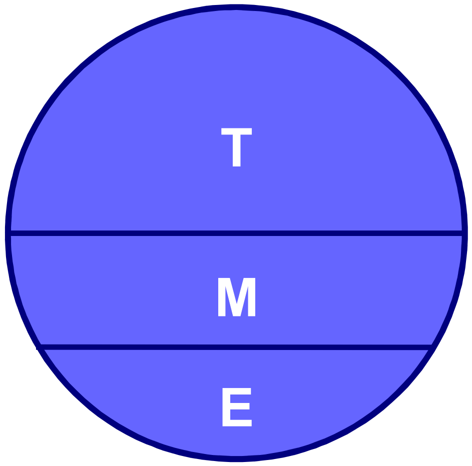
- The total variability of observed scores consists of both trait (substantive) variance and random error variance.
- This results in unreliability of measurement and the attenuation of observed correlations.

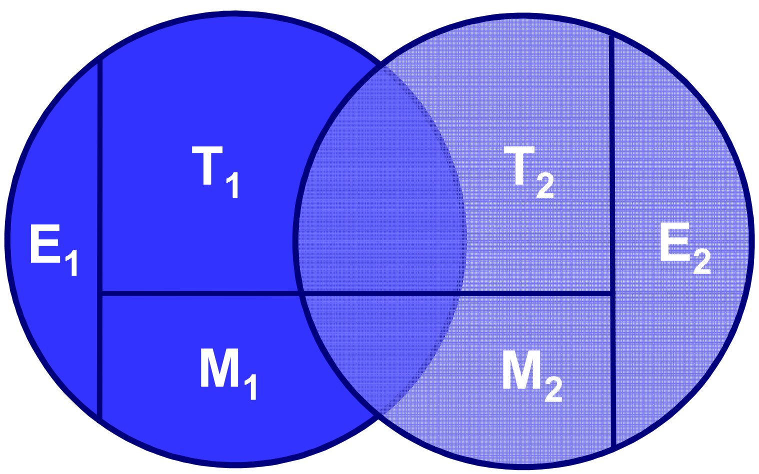
The relationship between observed measurements and constructs of interest (cont'd)

Solution: Use multiple indicators to measure the focal construct, in which case we can assess reliability and correct for attenuation.



The relationship between observed measurements and constructs of interest (cont'd)





- The total variability of observed scores consists of trait (substantive), random error, and systematic error (method) variance.
- This is likely to confound the assessment of reliability and relationships with other constructs.
- It also complicates the comparison of means.

A comprehensive model of measurement error

$$y_{ijt} = \tau_{ijt} + \lambda_{ijt}\,\eta_{jt} + \omega_{ijt} + \varepsilon_{ijt}$$

systematic error

random error

y_{ijt} $\rightarrow$ a person's observed score on the i^{th} measure of construct j at time t

η_{jt} $\rightarrow$ a person's unobserved score for construct j at time t

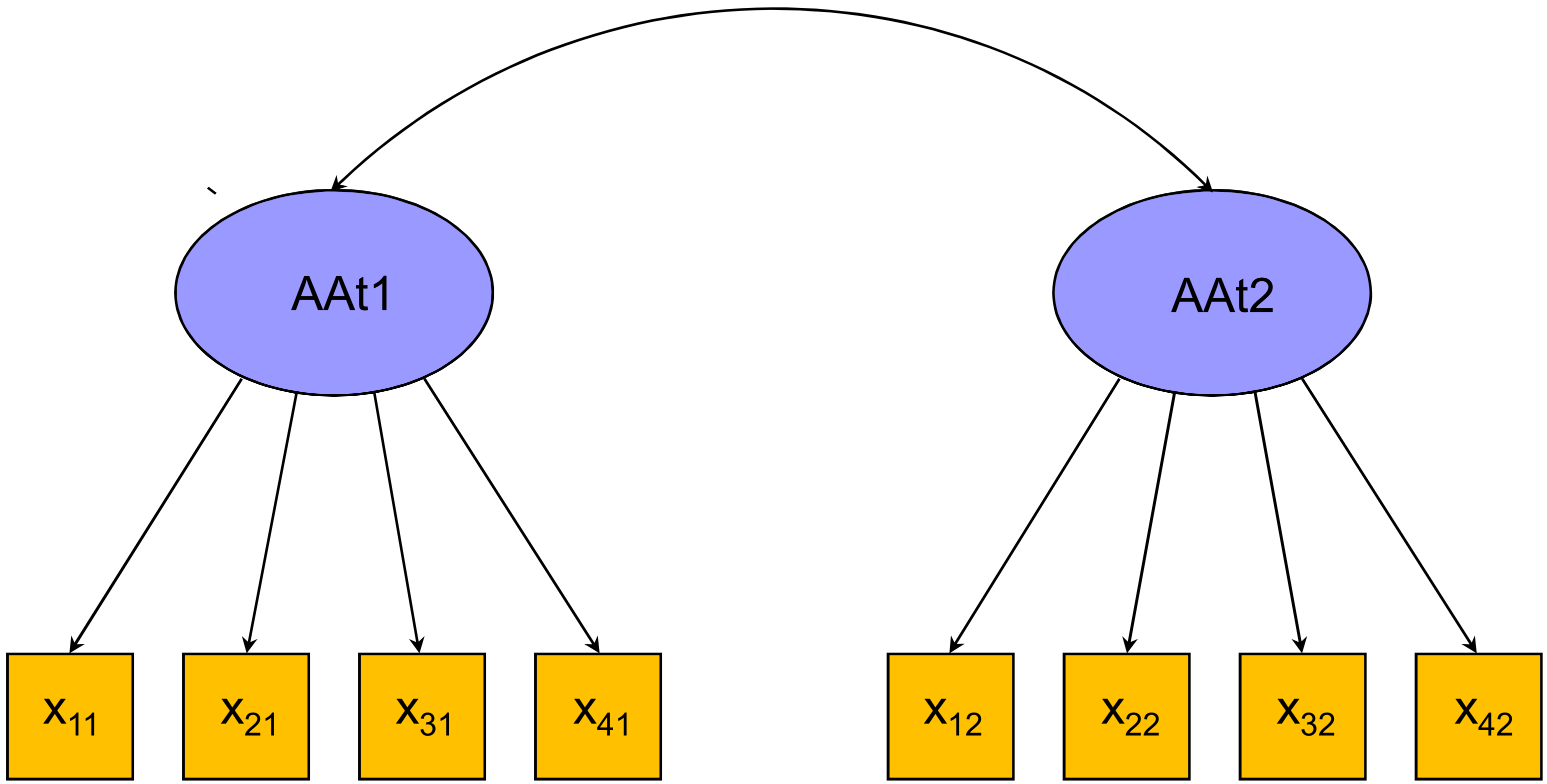
ω_{ijt} $\rightarrow$ systematic error score

ε_{ijt} $\rightarrow$ random error score

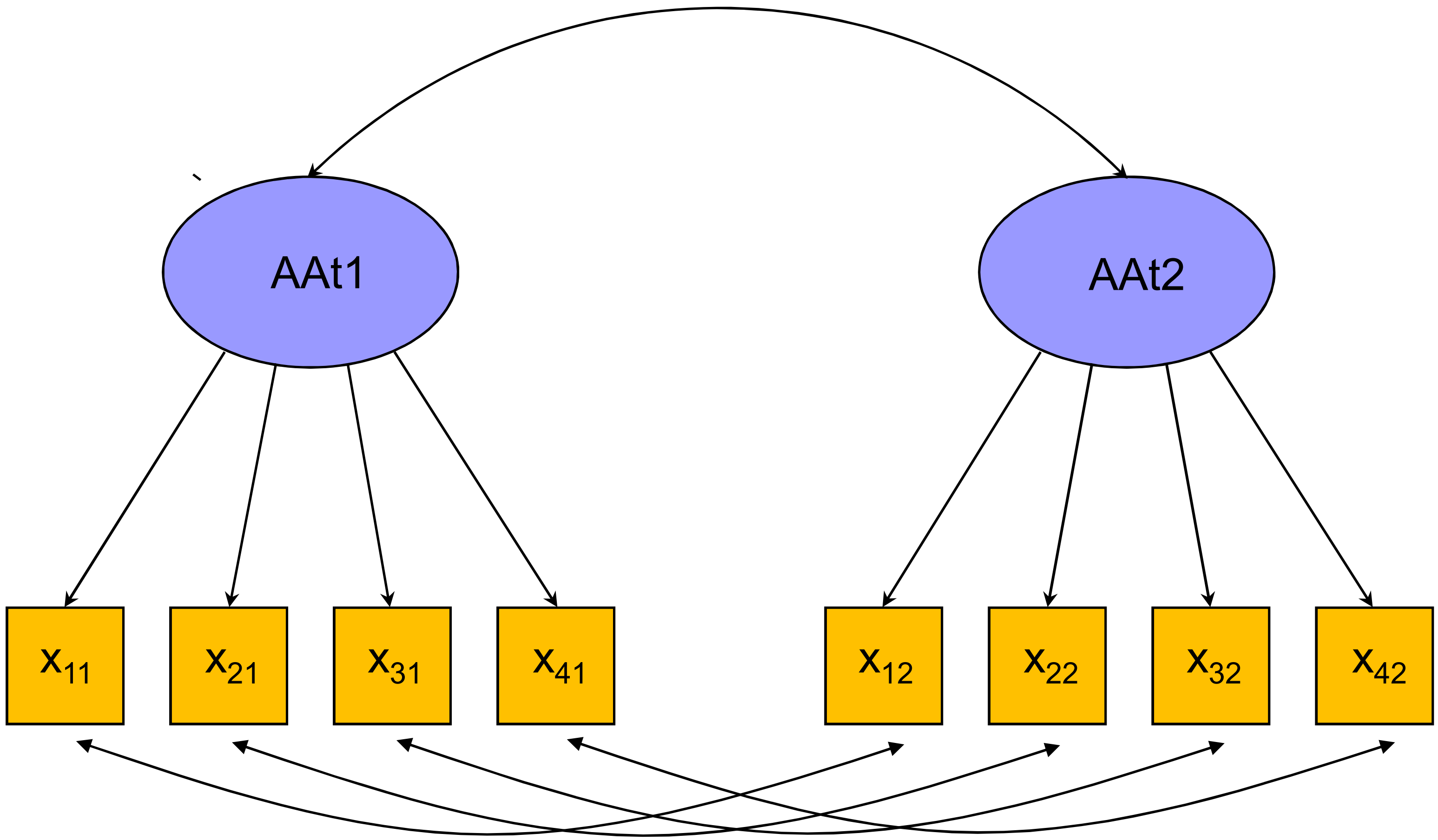
λ_{ijt} $\rightarrow$ coefficient (factor loading) relating y_{ijt} to η_{jt}

τ_{ijt} $\rightarrow$ intercept term (additive bias)

Attitude toward using coupons (measured at two points in time)



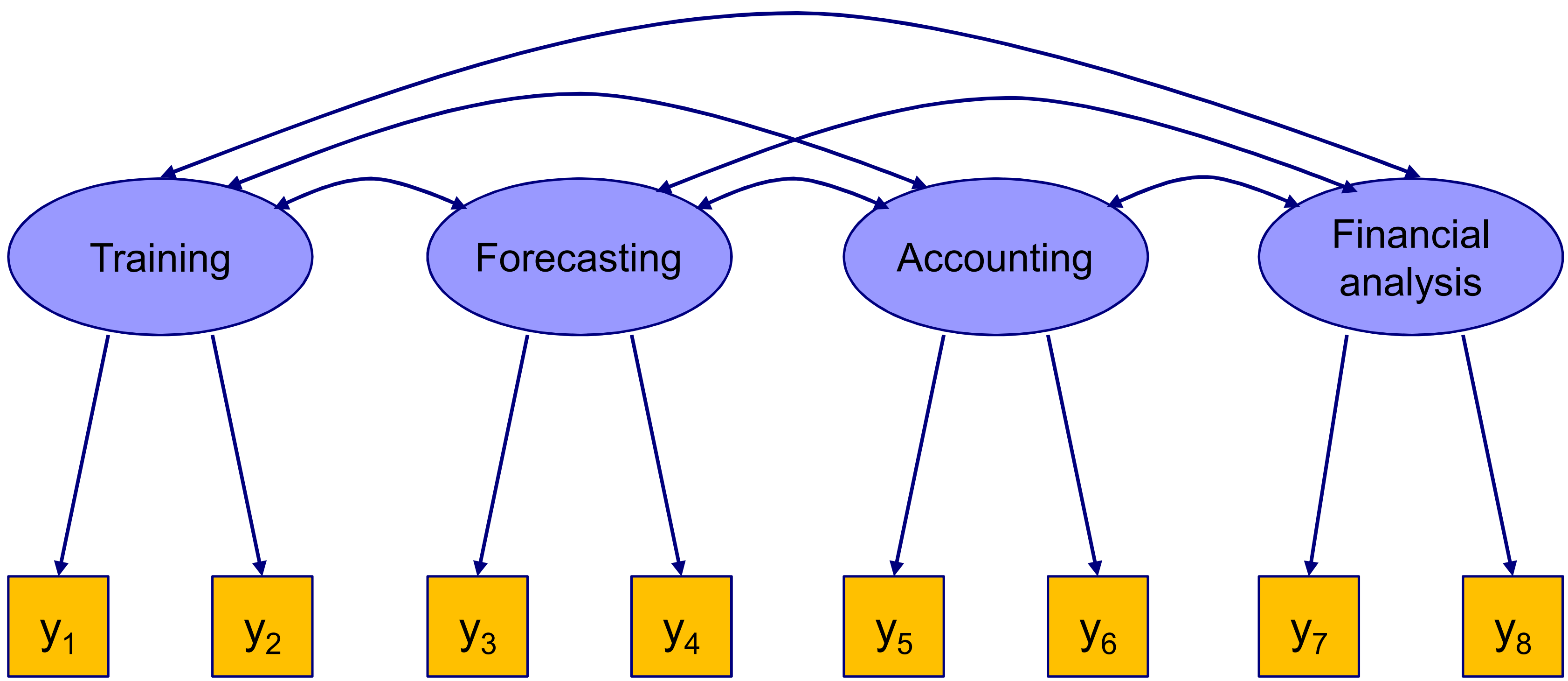
Attitude toward using coupons (measured at two points in time)



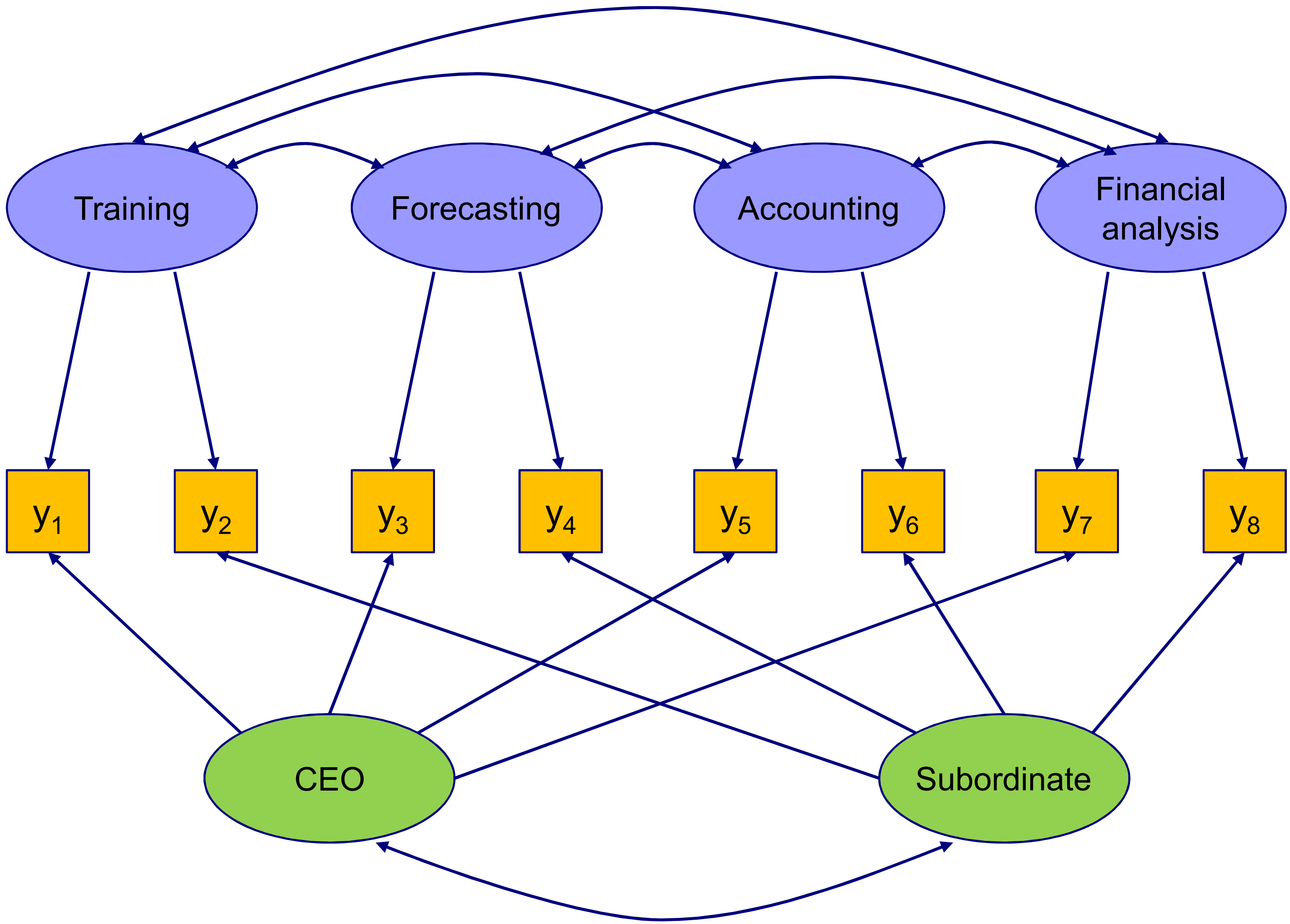
Factor correlations

	Original correlation	Corrected correlation
Exploratory factor analysis (PFA with Promax rotation)	.75	n.a.
Confirmatory factor analysis	.90	.90
Correlation of unweighted linear composites at t_1, t_2	.82	$\frac{.819}{\sqrt{.882}\sqrt{.911}} = .91$
Average correlation of individual t_1, t_2 measures	.63	$\frac{.626}{\sqrt{.654}\sqrt{.719}} = .91$

Adoption of managerial innovations (Bagozzi and Phillips 1982)



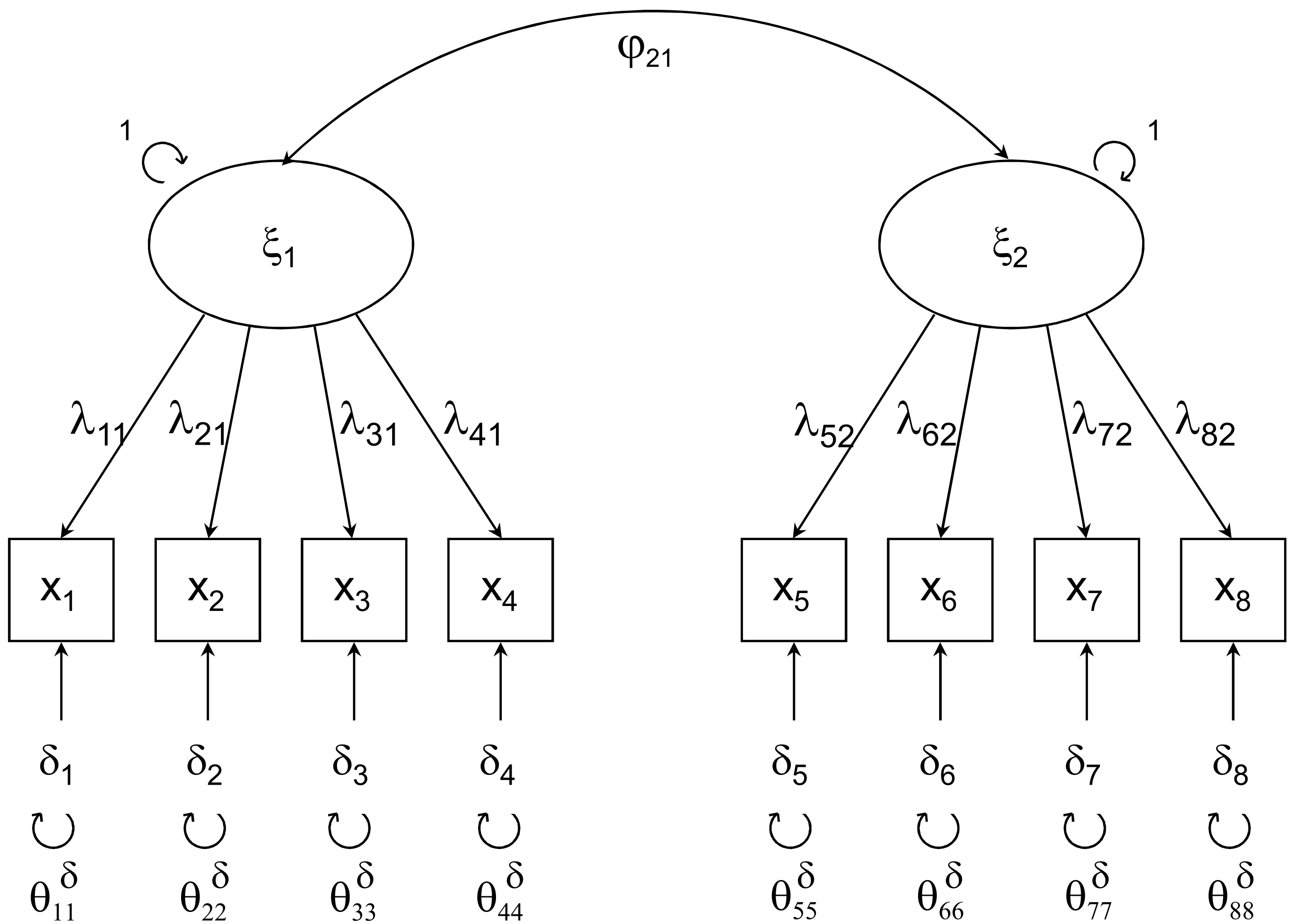
Adoption of managerial innovations (cont'd)



Variance partitioning

	Trait	Method	Error
Training-CEO (y_1)	.78	.07	.15
Training-Sub (y_2)	.25	.23	.53
Forecasting-CEO (y_3)	.90	.09	.00
Forecasting-Sub (y_4)	.25	.51	.23
Accounting-CEO (y_5)	.68	.14	.17
Accounting-Sub (y_6)	.93	.04	.03
Financial analysis-CEO (y_7)	.62	.38	.00
Financial analysis-Sub (y_8)	.74	.10	.15

Graphical specification of a (congeneric) measurement model



Need for Touch (NFT) scale (Peck and Childers 2003)

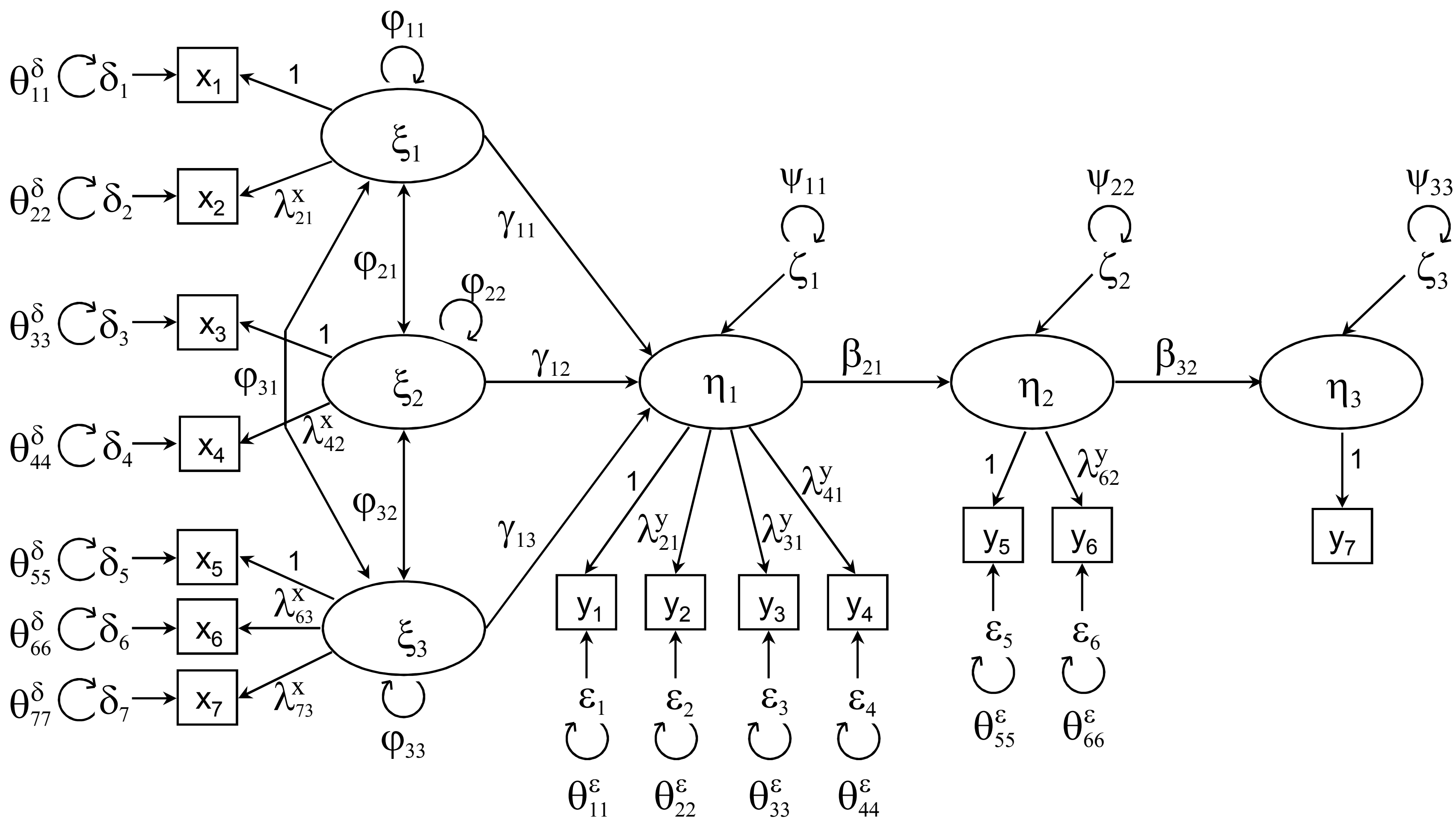
Instrumental touch:

- I place more trust in products that can be touched before purchase.
- I feel more comfortable purchasing a product after physically examining it.
- If I can't touch a product in the store, I am reluctant to purchase the product.
- I feel more confident making a purchase after touching a product.
- The only way to make sure a product is worth buying is to actually touch it.
- There are many products that I would only buy if I could handle them before purchase.

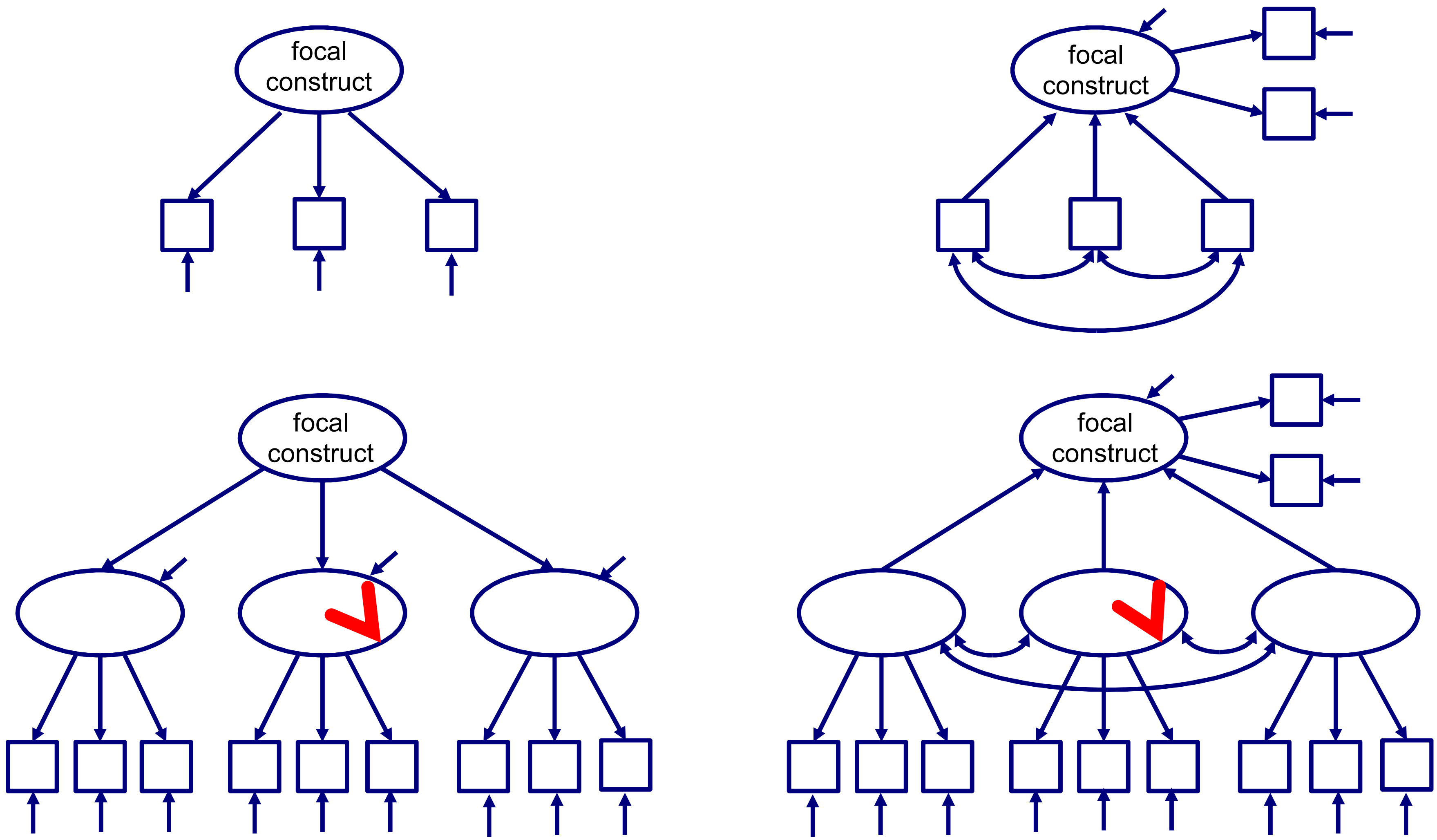
Autotelic touch:

- When walking through stores, I can't help touching all kinds of products.
- Touching products can be fun.
- When browsing in stores, it is important for me to handle all kinds of products.
- I like to touch products even if I have no intention of buying them.
- When browsing in stores, I like to touch lots of products.
- I find myself touching all kinds of products in stores.

Graphical specification of an integrated measurement/latent variable model

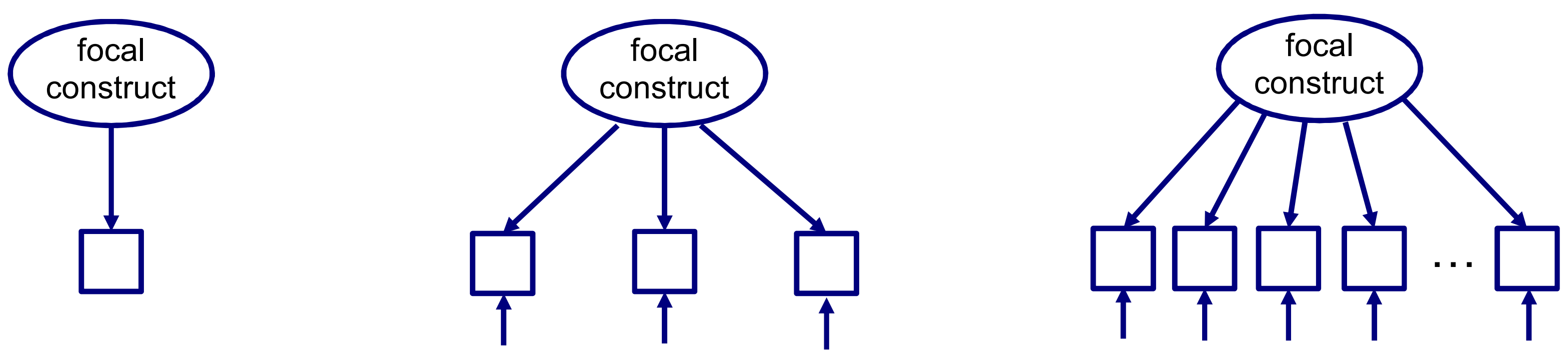


Measurement model specification issues: Reflective vs. formative measurement models



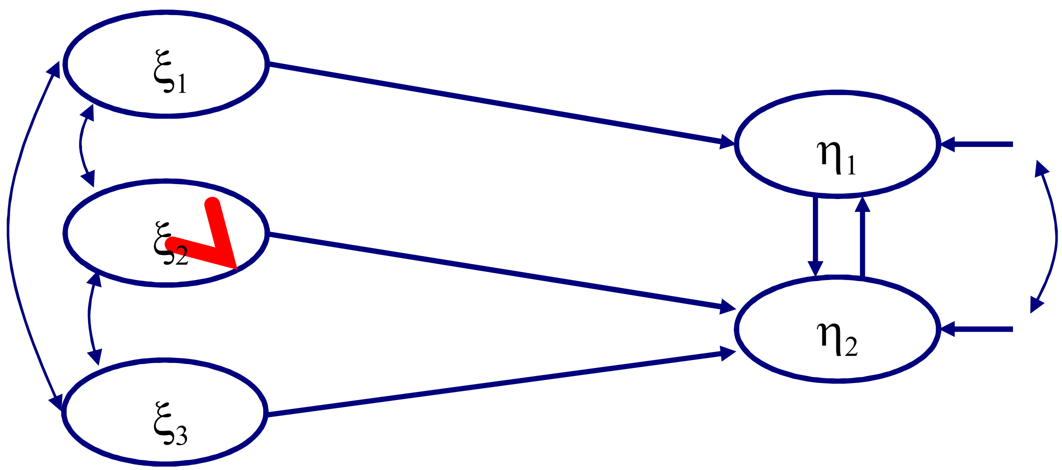
Measurement model specification issues: Number of indicators per construct

- in principle, more indicators are better, but there are practical limits;
- question of how explicitly single-item measures are modeled:
  - total aggregation model
  - partial aggregation model (item parcels)
  - total disaggregation model



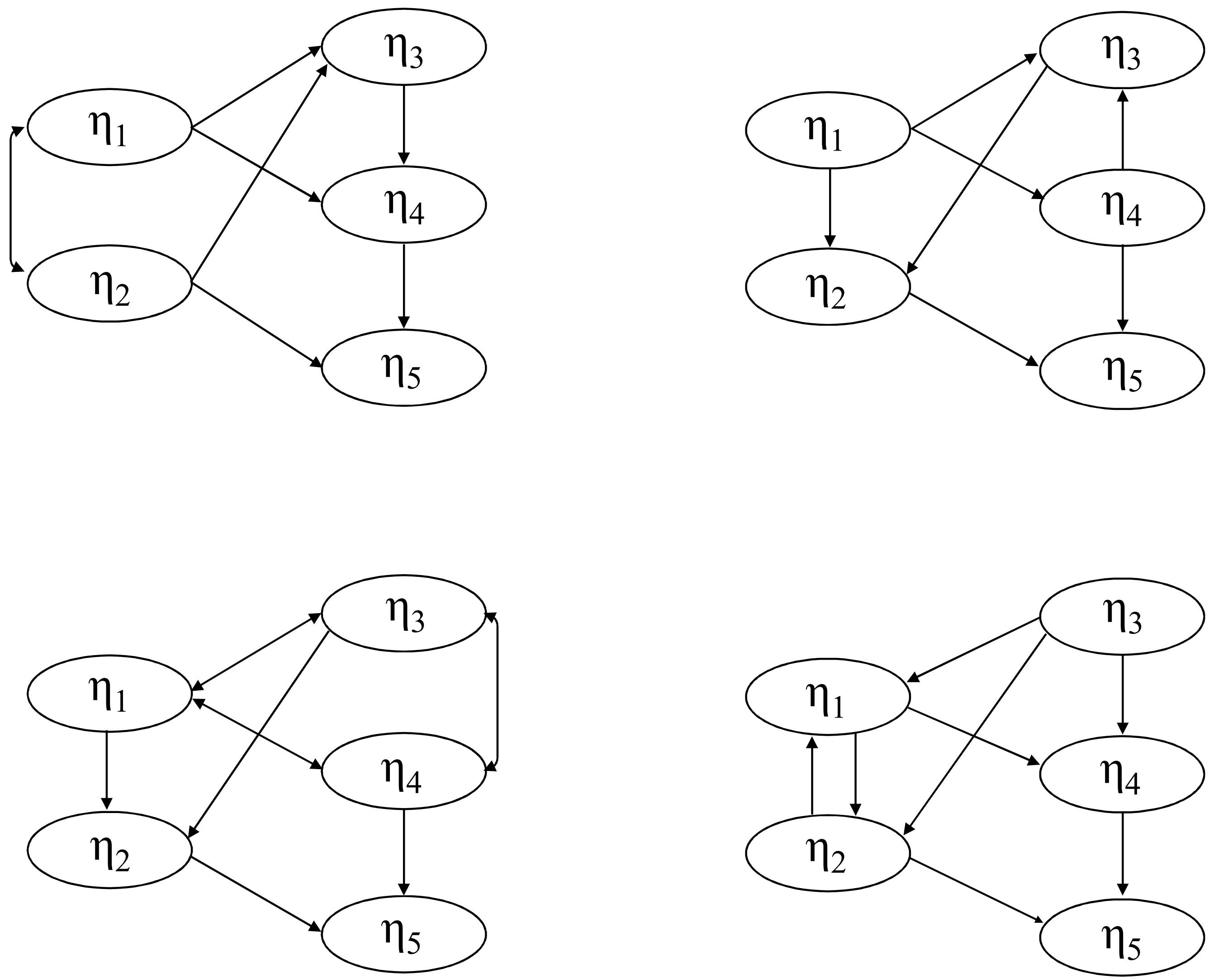
Latent variable model specification issues

- recursive vs. nonrecursive models



- specification of plausible alternative models
- problem of equivalent models

The problem of equivalent models



Model identification

- question whether the parameters in the model are uniquely determined so that the conclusions derived from the analysis aren't arbitrary;
- a necessary condition is that the number of parameters to be estimated doesn't exceed the number of unique elements in the (co)variance matrix of the observed variables;
- for relatively simple models, rules of identification are available; for more complex models, empirical heuristics may have to be used;

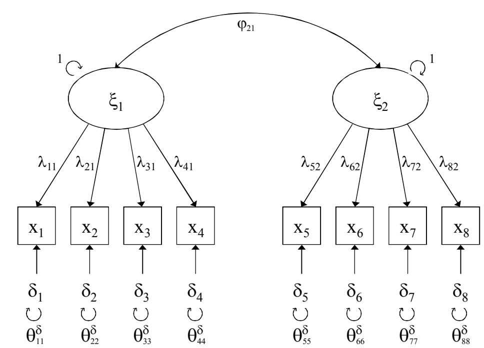
Model estimation

- Covariance-based SEM:
  - estimate the model parameters in such a way that the covariance matrix implied by the estimated parameters is as close as possible to the sample covariance matrix;

    e.g., for a factor model

$$x = \Lambda \xi + \delta$$

$$\Sigma = \Lambda \Phi \Lambda' + \Theta$$

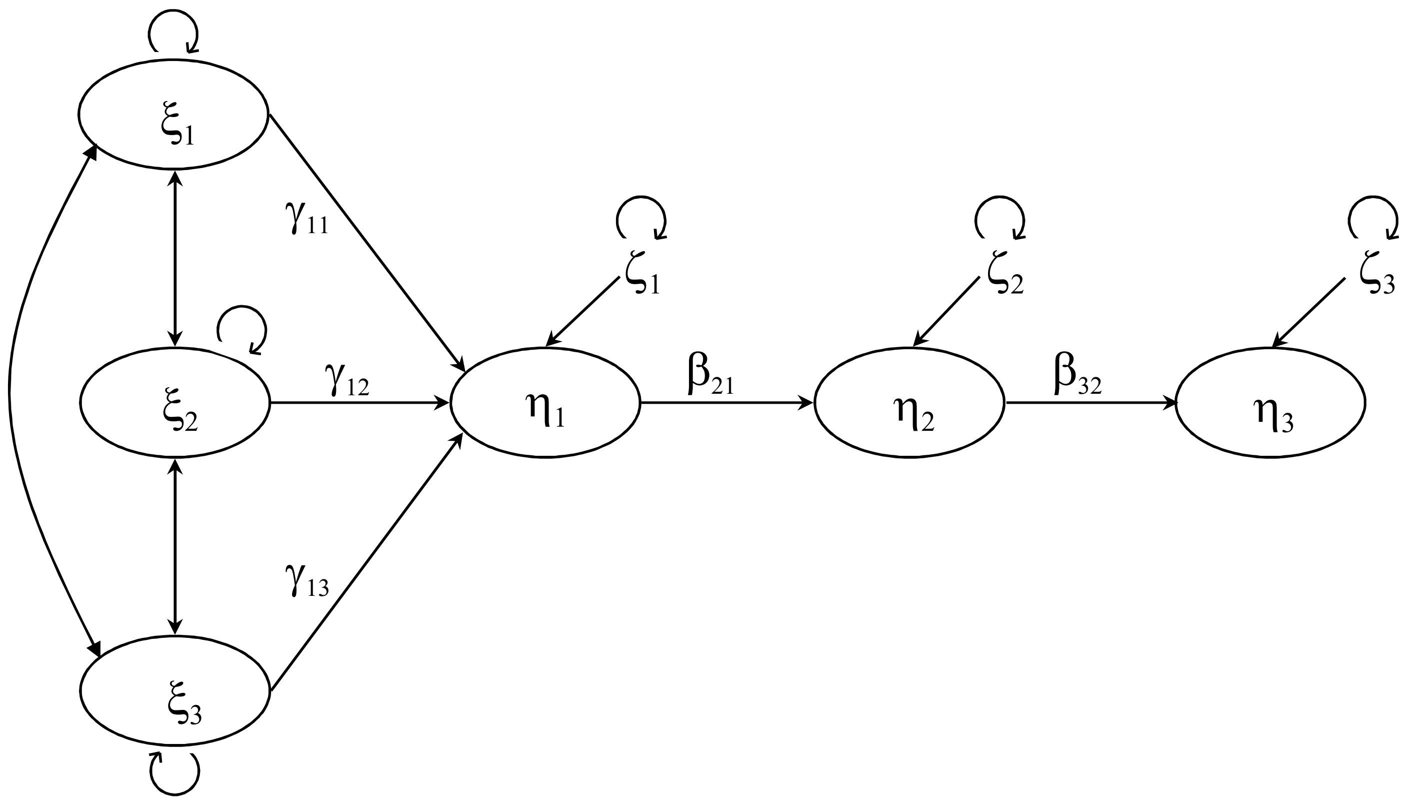


- Variance-based SEM (PLS):
  - estimate the parameters so as to maximize the explained variance in the dependent variables;

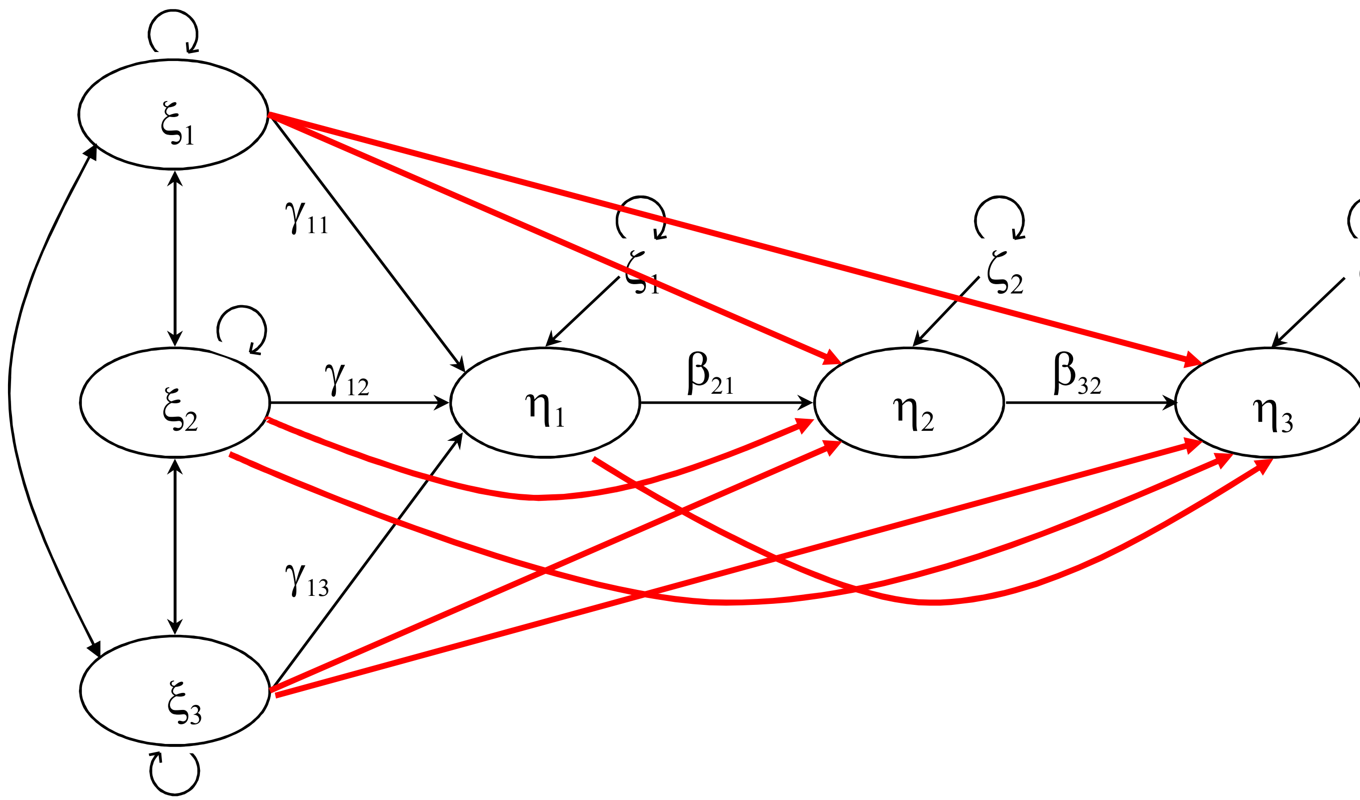
Model testing

- **Global fit measures**:
  - χ^2 goodness of fit test
  - alternative fit indices
- **Local fit measures**:
  - parameter estimates, standard errors and z-values
  - measurement model:
    - reliability and discriminant validity
  - latent variable model:
    - R^2 for each structural equation
- **Model modification**:
  - modification indices and EPC's
  - residuals

Testing the overidentifying restrictions of a model

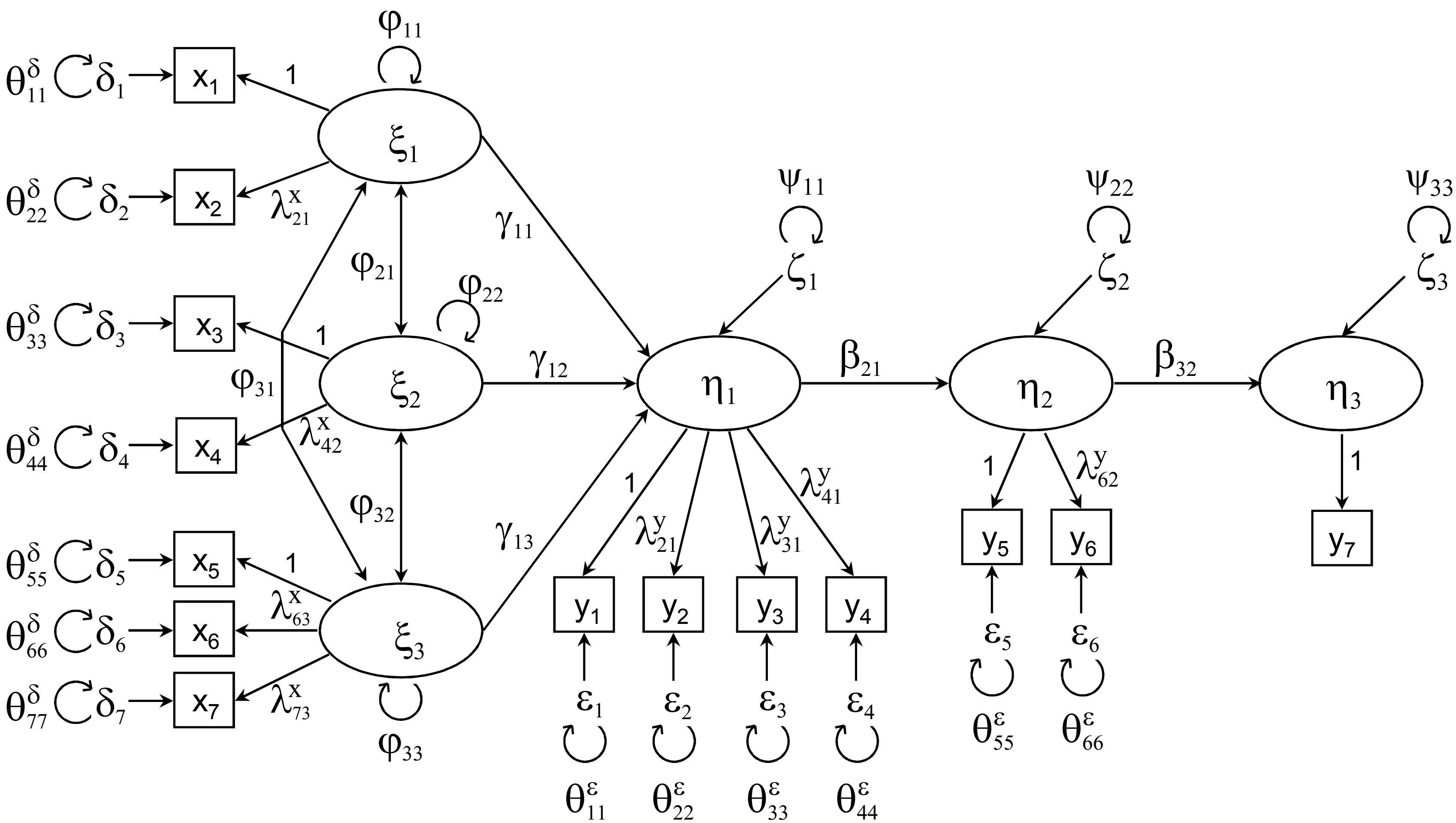


Testing the overidentifying restrictions of a model (cont'd)

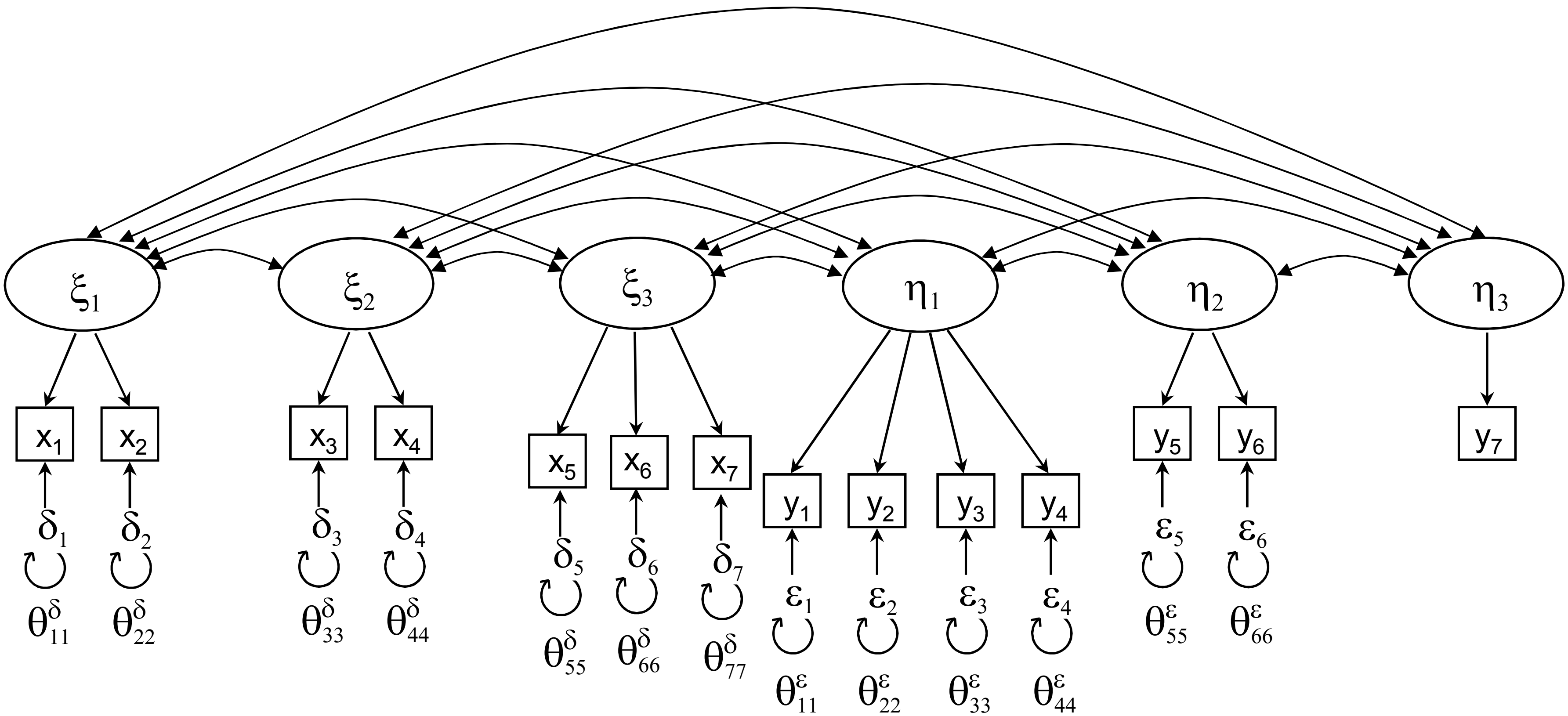


There are 21 distinct elements in the covariance matrix of the 6 latent variables, we estimate 14 parameters, so there are 7 overidentifying restrictions.

Testing the overidentifying restrictions of a model (cont'd)

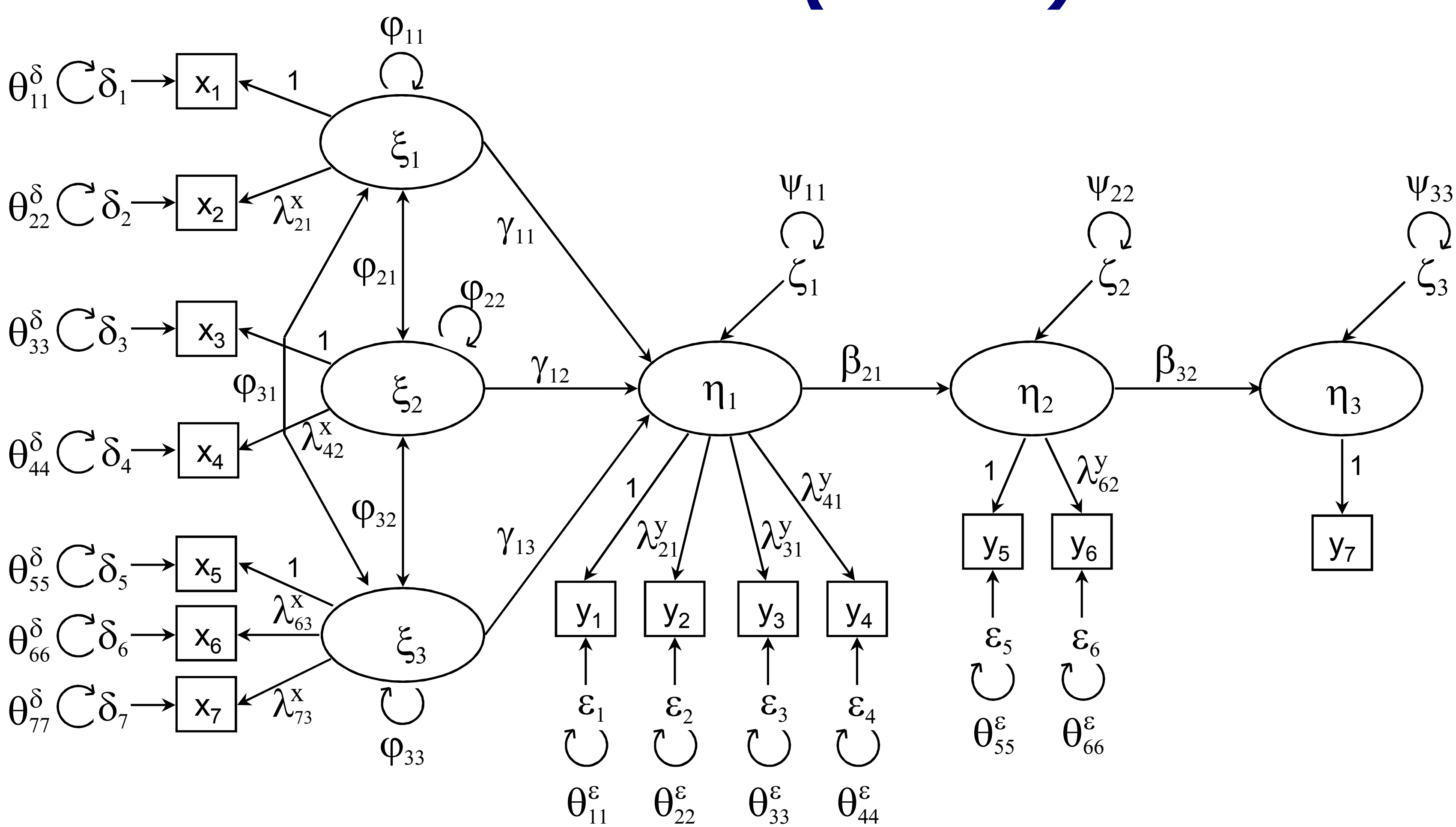


Testing the overidentifying restrictions of a model (cont'd)



There are 105 distinct elements in the covariance matrix of the 14 observed variables, we estimate 42 parameters, so there are 63 overidentifying restrictions.

Testing the overidentifying restrictions of a model (cont'd)



There are 105 distinct elements in the covariance matrix of the 14 observed variables, we estimate 35 parameters, so there are 70 (63+7) overidentifying restrictions.

Problems with the χ^2 test

- it is not a robust test;
- it is based on the accept-support logic of testing:
  - a model is more likely to get support when the sample size is small and power is low (even though it is an asymptotic test);
  - since most models are unlikely to be literally true in the population, in large samples the model is likely to be rejected;
- thus, many alternative fit indices have been suggested;

Classification of overall fit indices

Overall fit indices

- Stand-alone fit indices
  - χ^2 test and variations
    - minimum fit function χ^2 (C1)
    - normal theory WLS χ^2 (C2)
    - S-B scaled χ^2 (C3)
    - χ^2 corrected for non-normality (C4)
    - χ^2/df
    - minimum fit function f
    - Scaled LR
  - Noncentrality-based measures
    - NCP
    - Rescaled NCP (t)
    - RMSEA
    - MC
  - Information theory-based measures
    - AIC
    - SBC
    - CIC
    - ECVI
  - Others
    - (S)RMR
    - GFI
    - PGFI
    - AGFI
    - Gamma hat
    - CN
- Incremental fit indices
  - Type I indices
  - Type II indices

Type I indices		Type II indices
NFI	[χ^2 or f]	IFI
RFI	[χ^2/df]	TLI
	CFI [χ^2-df]	
	TLI [(χ^2-df)/df]	

Goodness of fit statistics for the coupon data:

Degrees of Freedom = 70
Minimum Fit Function Chi-Square = 93.63 (P = 0.031)
Normal Theory Weighted Least Squares Chi-Square = 92.60 (P = 0.037)
Estimated Non-centrality Parameter (NCP) = 22.60
90 Percent Confidence Interval for NCP = (1.60 ; 51.68)

Minimum Fit Function Value = 0.38
Population Discrepancy Function Value (F0) = 0.091
90 Percent Confidence Interval for F0 = (0.0064 ; 0.21)
Root Mean Square Error of Approximation (RMSEA) = 0.036
90 Percent Confidence Interval for RMSEA = (0.0096 ; 0.054)
P-Value for Test of Close Fit (RMSEA < 0.05) = 0.89

Expected Cross-Validation Index (ECVI) = 0.65
90 Percent Confidence Interval for ECVI = (0.57 ; 0.77)
ECVI for Saturated Model = 0.84
ECVI for Independence Model = 12.16

Chi-Square for Independence Model with 91 Degrees of Freedom = 2999.42
Independence AIC = 3027.42
Model AIC = 162.60
Saturated AIC = 210.00
Independence CAIC = 3090.72
Model CAIC = 320.85
Saturated CAIC = 684.75

Normed Fit Index (NFI) = 0.97
Non-Normed Fit Index (NNFI) = 0.99
Parsimony Normed Fit Index (PNFI) = 0.75
Comparative Fit Index (CFI) = 0.99
Incremental Fit Index (IFI) = 0.99
Relative Fit Index (RFI) = 0.96

Critical N (CN) = 268.08

Root Mean Square Residual (RMR) = 0.13
Standardized RMR = 0.049
Goodness of Fit Index (GFI) = 0.95
Adjusted Goodness of Fit Index (AGFI) = 0.92
Parsimony Goodness of Fit Index (PGFI) = 0.63

Model testing

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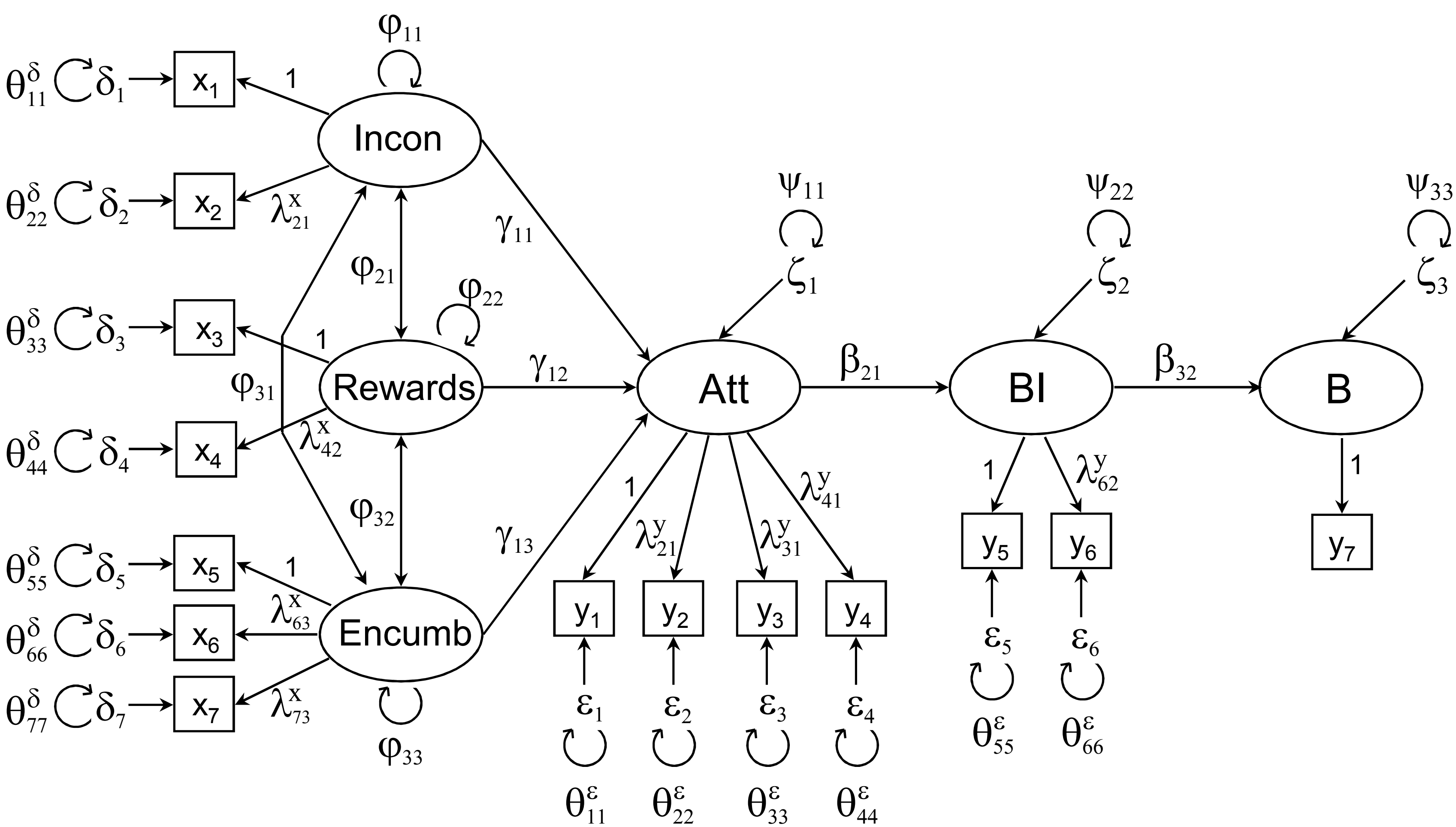
Estimation results for the coupon model



Measurement model results for coupon data

Construct	parameter	parameter estimate	standardized parameter estimate	z-value	individual-item reliability	composite reliability (average variance extracted)
Inconveniences						.88 (.78)
	λ^x_{11}	1.00	0.89	--	0.79	
	λ^x_{21}	0.98	0.88	11.32	0.77	
	θ^δ_{11}	0.56	0.21	3.32	--	
	θ^δ_{22}	0.61	0.23	3.71	--	
Rewards						.76 (.61)
	λ^x_{32}	1.00	0.86	--	0.75	
	λ^x_{42}	0.82	0.70	6.89	0.48	
	θ^δ_{33}	0.45	0.25	2.55	--	
	θ^δ_{44}	0.96	0.52	6.63	--	
Encumbrances						.70 (.45)
	λ^x_{53}	1.00	0.49	--	0.24	
	λ^x_{63}	1.73	0.77	6.30	0.59	
	λ^x_{73}	1.48	0.71	6.30	0.50	
	θ^δ_{55}	2.78	0.76	9.97	--	
	θ^δ_{66}	1.85	0.41	5.49	--	
	θ^δ_{77}	1.92	0.50	6.87	--	

Measurement model results for coupon data (cont'd)

Construct	parameter	parameter estimate	standardized parameter estimate	z-value	individual-item Reliability	composite reliability (average variance extracted)
Attitudes						.88 (.66)
	λ^y_{11}	1.00	0.80	--	0.63	
	λ^y_{21}	1.04	0.86	14.97	0.74	
	λ^y_{31}	0.85	0.73	12.14	0.53	
	λ^y_{41}	1.10	0.84	14.58	0.71	
	θ^ε_{11}	0.68	0.37	9.06	--	
	θ^ε_{22}	0.44	0.26	7.70	--	
	θ^ε_{33}	0.76	0.47	9.82	--	
	θ^ε_{44}	0.59	0.29	8.20	--	
Intentions						.91 (.84)
	λ^y_{42}	1.00	0.87	--	0.75	
	λ^y_{52}	1.09	0.97	18.91	0.93	
	θ^ε_{44}	0.97	0.25	7.04	--	
	θ^ε_{55}	0.25	0.07	1.95	--	
Behavior						
	λ^y_{63}	1.00	1.00	--	1.00	
	θ^ε_{66}	0.00	0.00	--	--	

Discriminant validity

Correlation Matrix of ETA and KSI

		aact (.81)	bi (.92)	bh (--)	inconv (.88)	rewards (.78)	encumbr (.67)
aact	(.81)	1.00					
bi	(.92)	0.70	1.00				
bh	(---)	0.40	0.58	1.00			
inconv	(.88)	-0.44	-0.31	-0.18	1.00		
rewards	(.78)	0.52	0.36	0.21	-0.10	1.00	
encumbr	(.67)	-0.35	-0.25	-0.14	0.49	-0.27	1.00

Note: The latent variable correlations are corrected for attenuation.

Latent variable model results for coupon data

Structural Equations

```
AACT =  - 0.28*INCONV + 0.44*REWARDS - 0.050*ENCUMBR, Errorvar.= 0.69 , R² = 0.42
         (0.058)       (0.081)        (0.097)                        (0.11)
         -4.77          5.42          -0.51                           6.52

  BI = 1.10*AACT, Errorvar.= 1.53 , R² = 0.48
      (0.11)                (0.20)
       10.04                 7.73

  BH = 0.49*BI, Errorvar.= 1.41 , R² = 0.34
      (0.049)             (0.13)
       10.10               10.78
```

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Modification indices

- a modification index (MI) refers to the predicted decrease of the χ^2 statistic when a fixed parameter is freely estimated or an equality constraint is relaxed;
- associated with each MI is an expected parameter change (EPC), which shows the predicted value of the freely estimated parameter;
- data-based model modifications have to be done carefully;

Modification indices for coupon data

Modification Indices for BETA

	AACT	BI	BH
AACT	- -	11.05	1.52
BI	- -	- -	2.34
BH	2.34	- -	- -

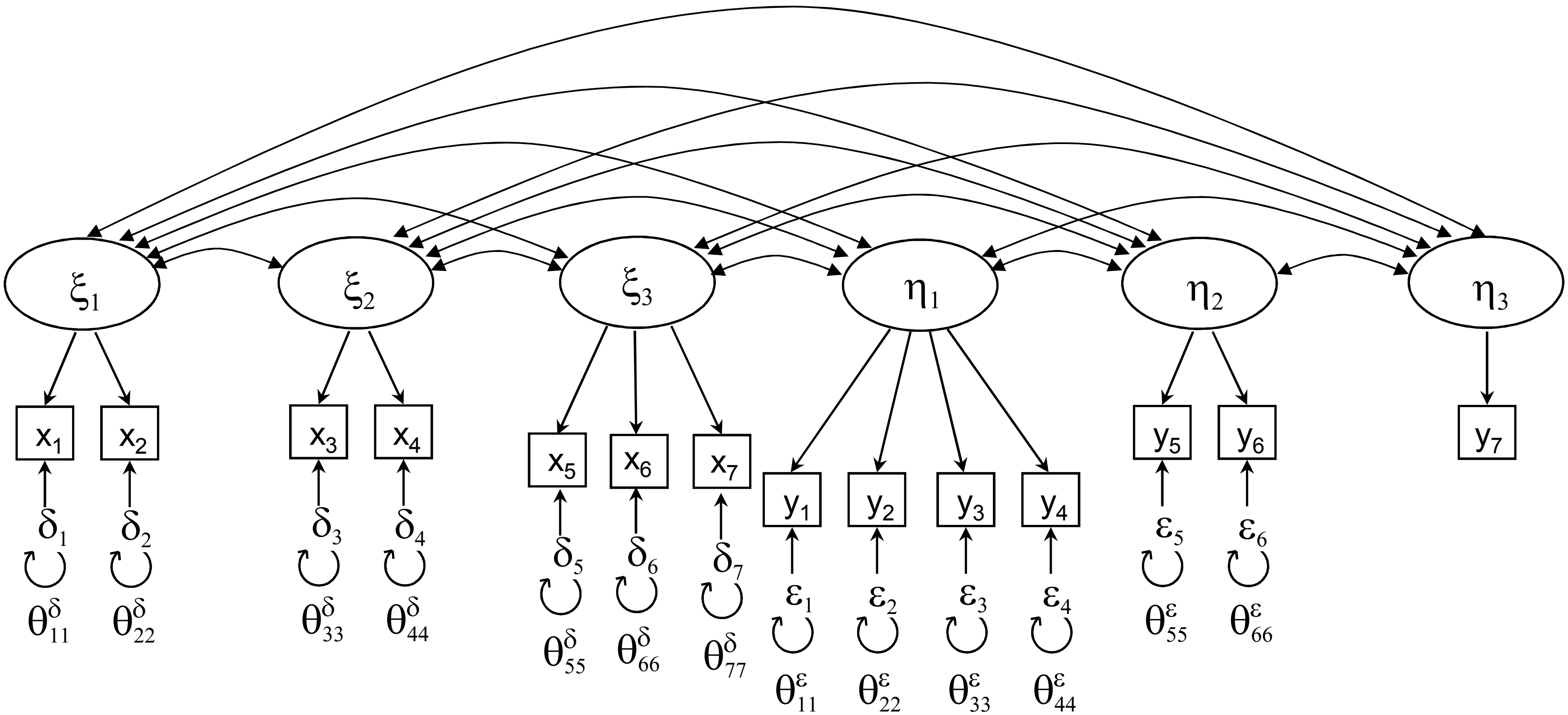
Modification Indices for GAMMA

	INCONV	REWARDS	ENCUMBR
AACT	- -	- -	- -
BI	5.57	3.07	5.15
BH	1.61	12.67	2.78

Two-step approach to model modification (Anderson and Gerbing 1988)

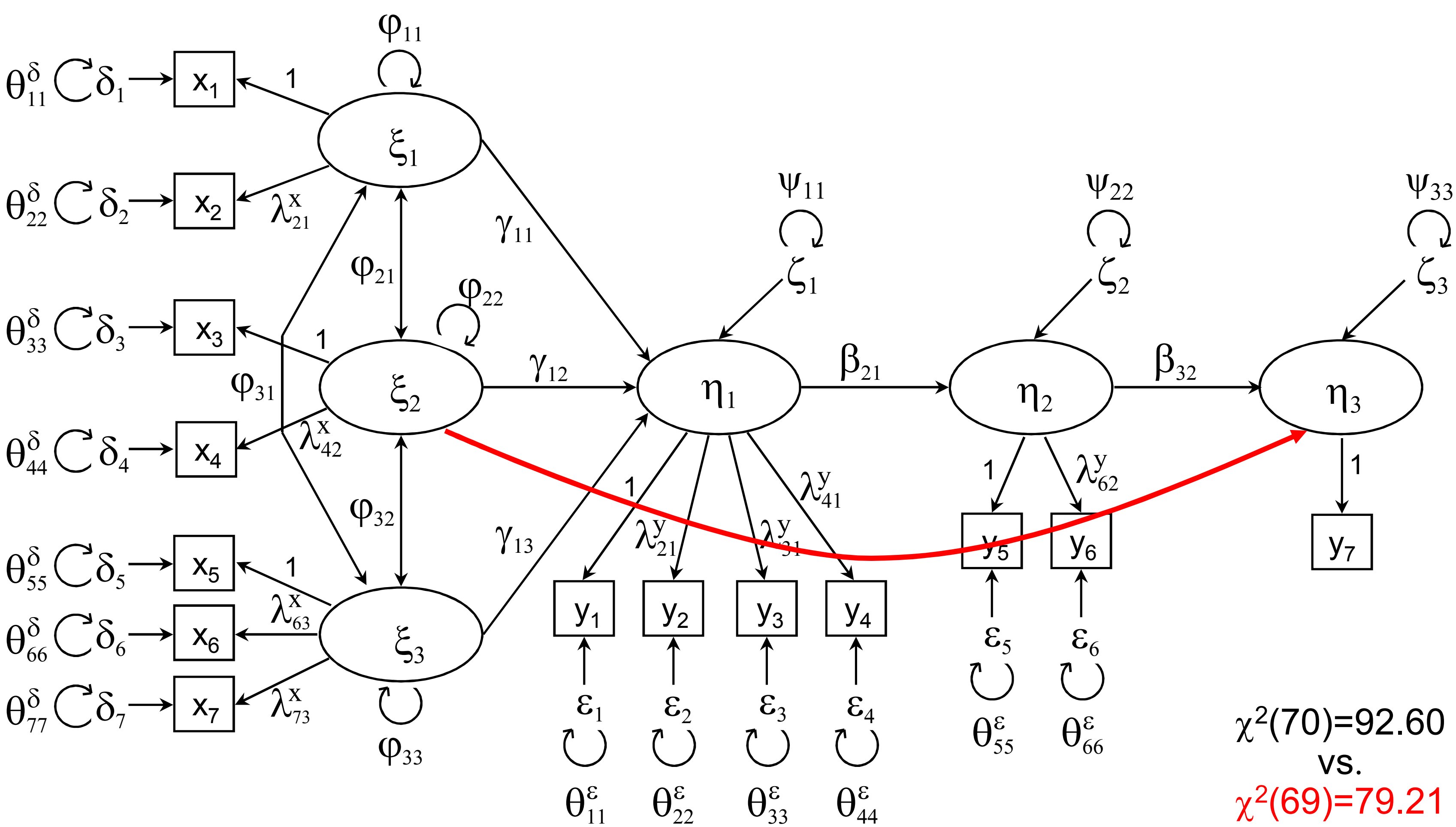
- specify a measurement model in which the latent variable model is saturated and purify the measurement model;
- once the measurement model is in place, attend to the latent variable model;

Saturated latent variable model for the coupon data



$\chi^2(63)=62.90$

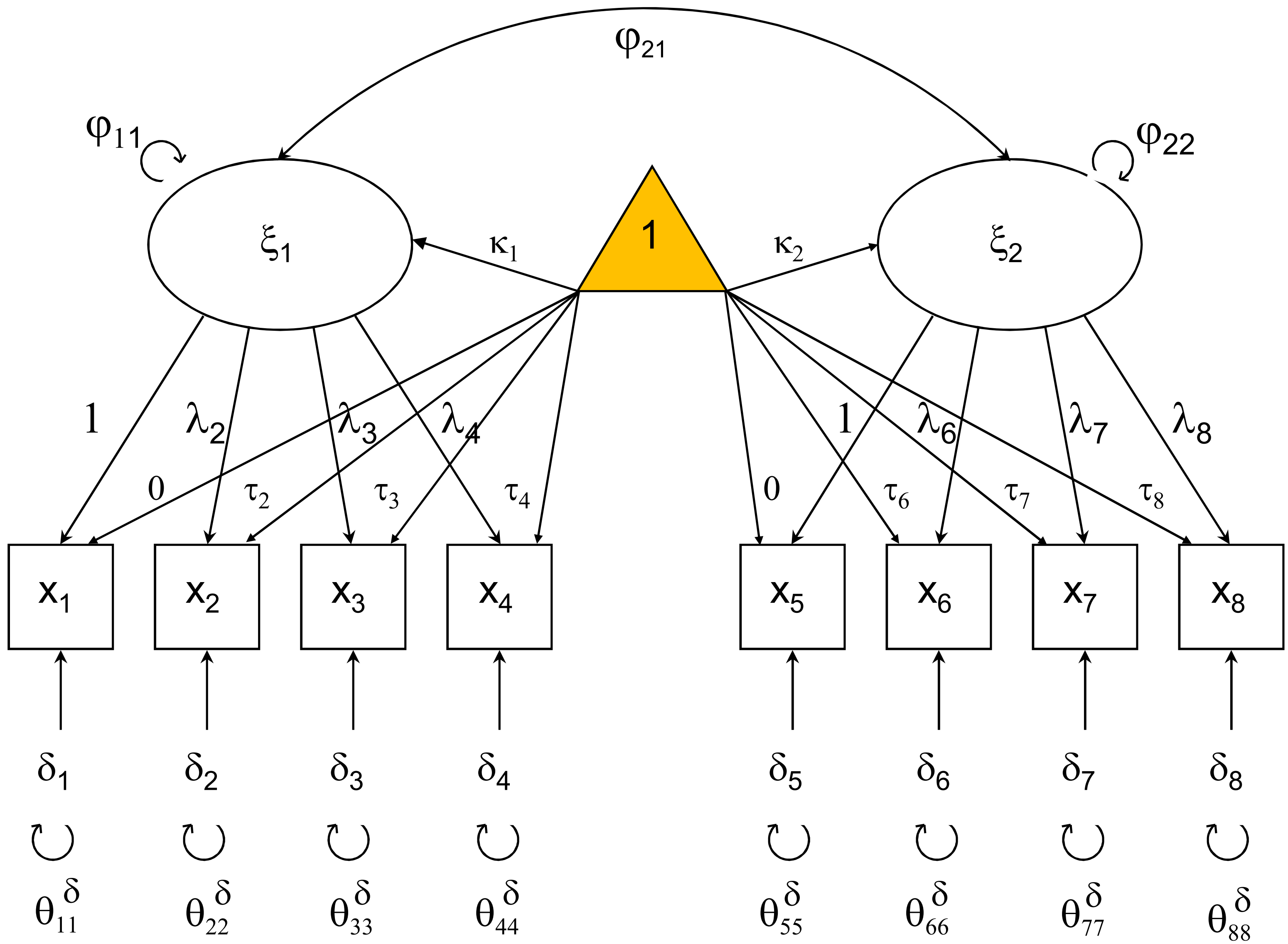
Modified latent variable model



Multi-sample analysis: Known population heterogeneity

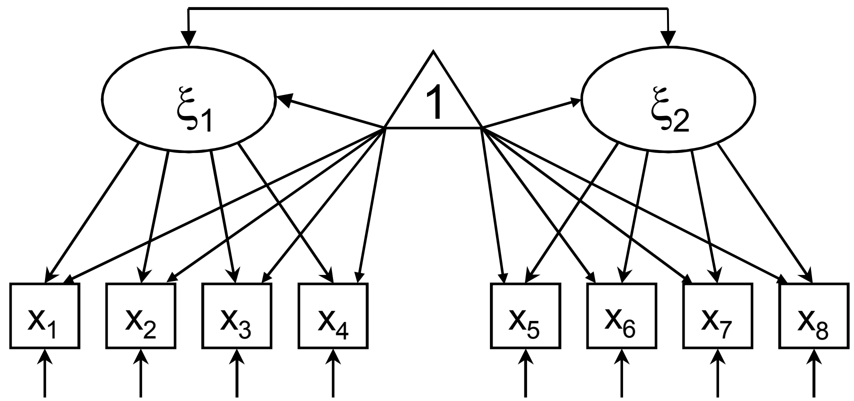
- SEM's can be specified for several populations simultaneously;
- this also allows the estimation of mean structures;
- multi-sample models are particularly useful for assessing measurement invariance (e.g., in cross-cultural research);
- mediation, moderation, moderated mediation and mediated moderation can be assessed in a straightforward fashion;

A factor model with a mean structure

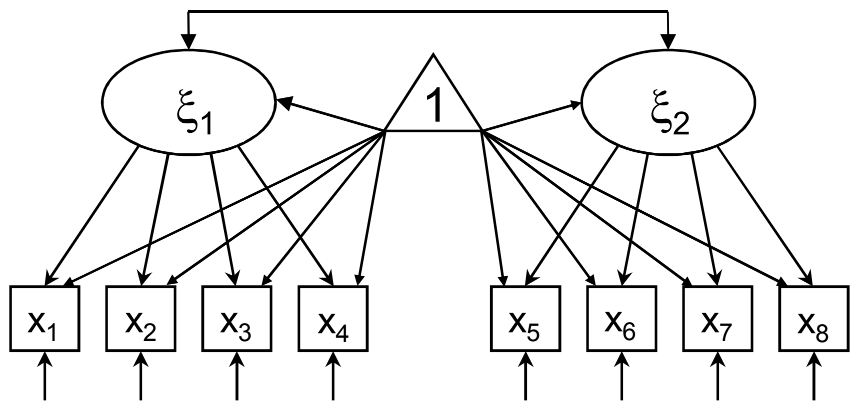


Assessing measurement invariance: Configural invariance

G1:

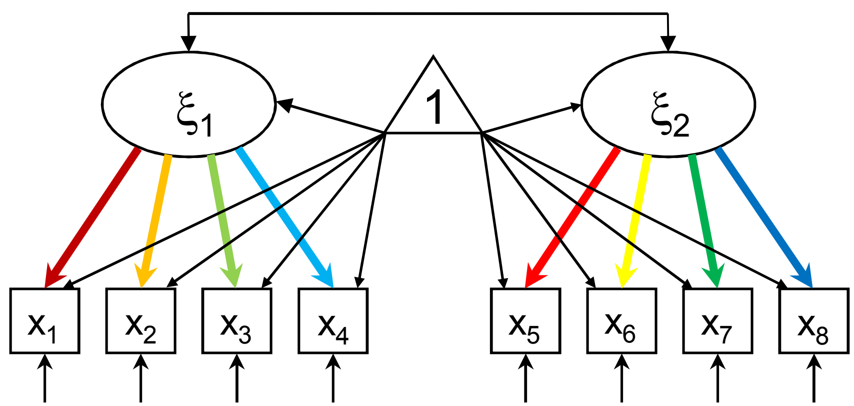


G2:

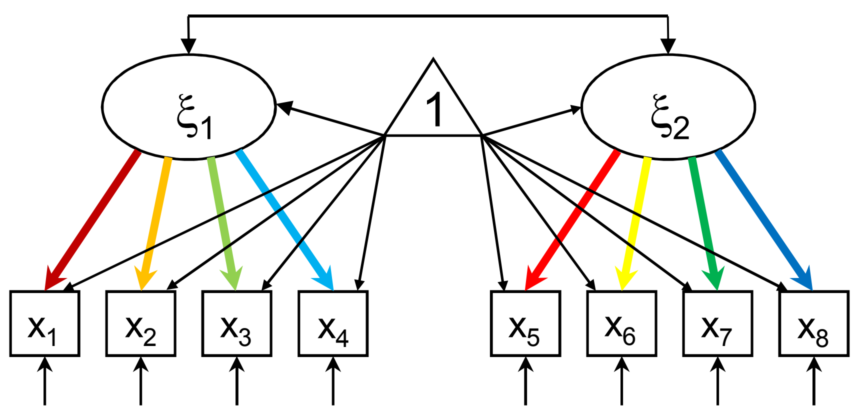


Assessing measurement invariance: Metric invariance

G1:

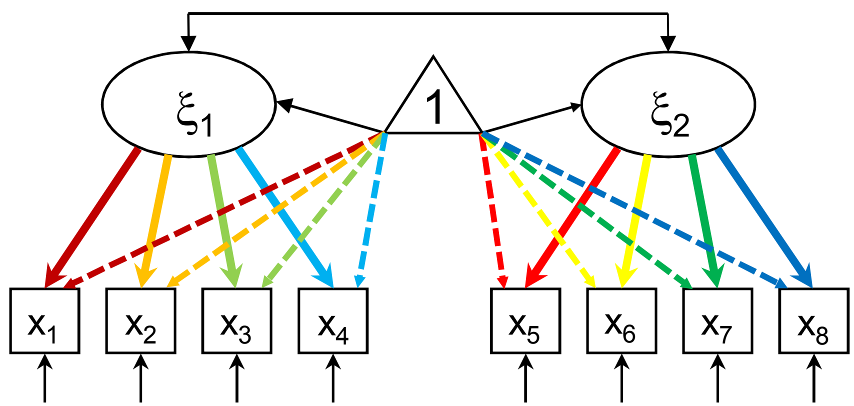


G2:

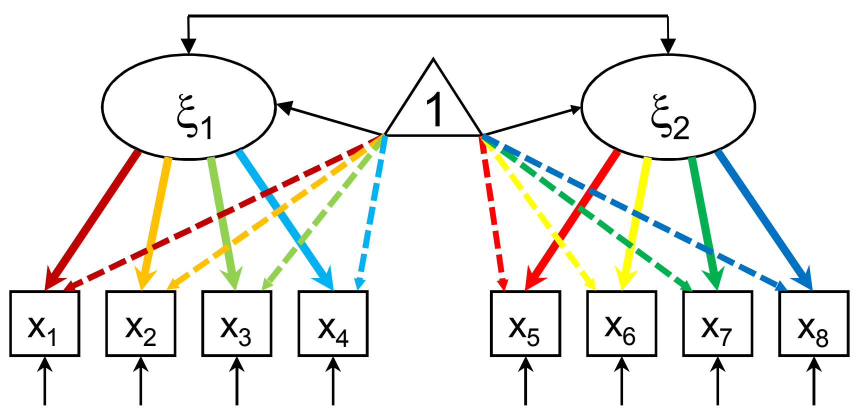


Assessing measurement invariance: Scalar invariance

G1:



G2:



Linking the types of invariance required to the research objective (Steenkamp and Baumgartner 1998)

	Configural invariance	Metric invariance	Scalar invariance
Exploring the basic structure of the construct cross-nationally	✓		
Examining structural relationships with other constructs cross-nationally	✓	✓	
Conducting cross-national comparisons of means	✓	✓	✓

Satisfaction with Life in the US and AUT: Final partial scalar invariance model

	Factor loadings		**Item intercepts**	
	AUT	US	AUT	US
ls1	.92	.92	-.03	-.03
ls2	.90	.90	.12	.12
ls3	1.00	1.00	0.00	0.00
ls4	.80	.80	.72	.72
ls5	1.10	.83	-1.00	.06

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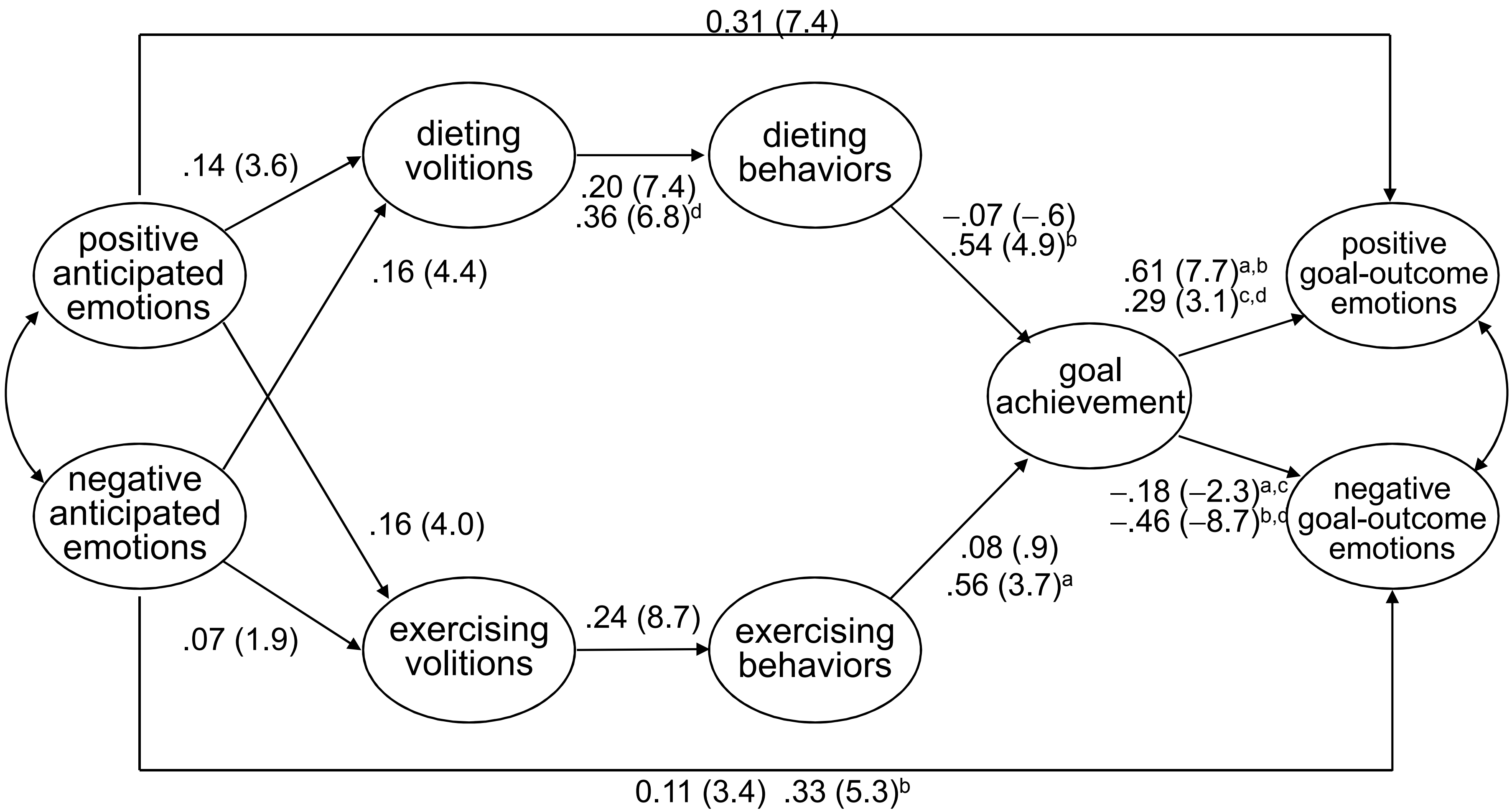
Latent means	AUT: 3.91		US: 3.26	

Goal-directed emotions: Results

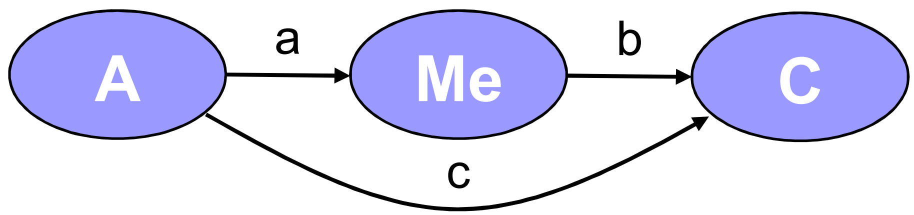


a men wanting to lose weight
b women wanting to lose weight
c men wanting to maintain their weight
d women wanting to maintain their weight

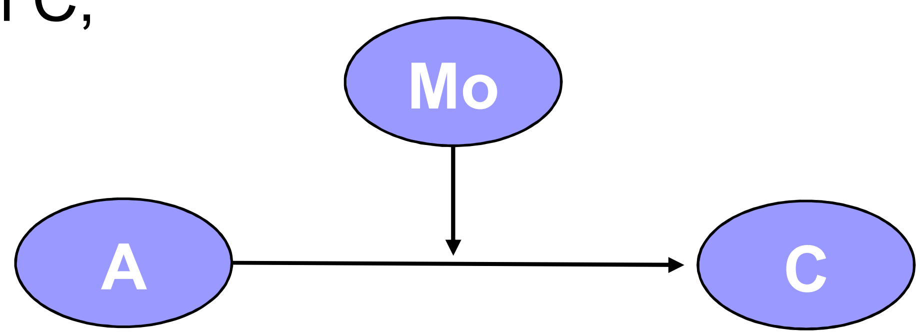
$\chi^2(110)=150.51$
RMSEA=.061
CFI=.94
TLI=.92

Mediation and moderation

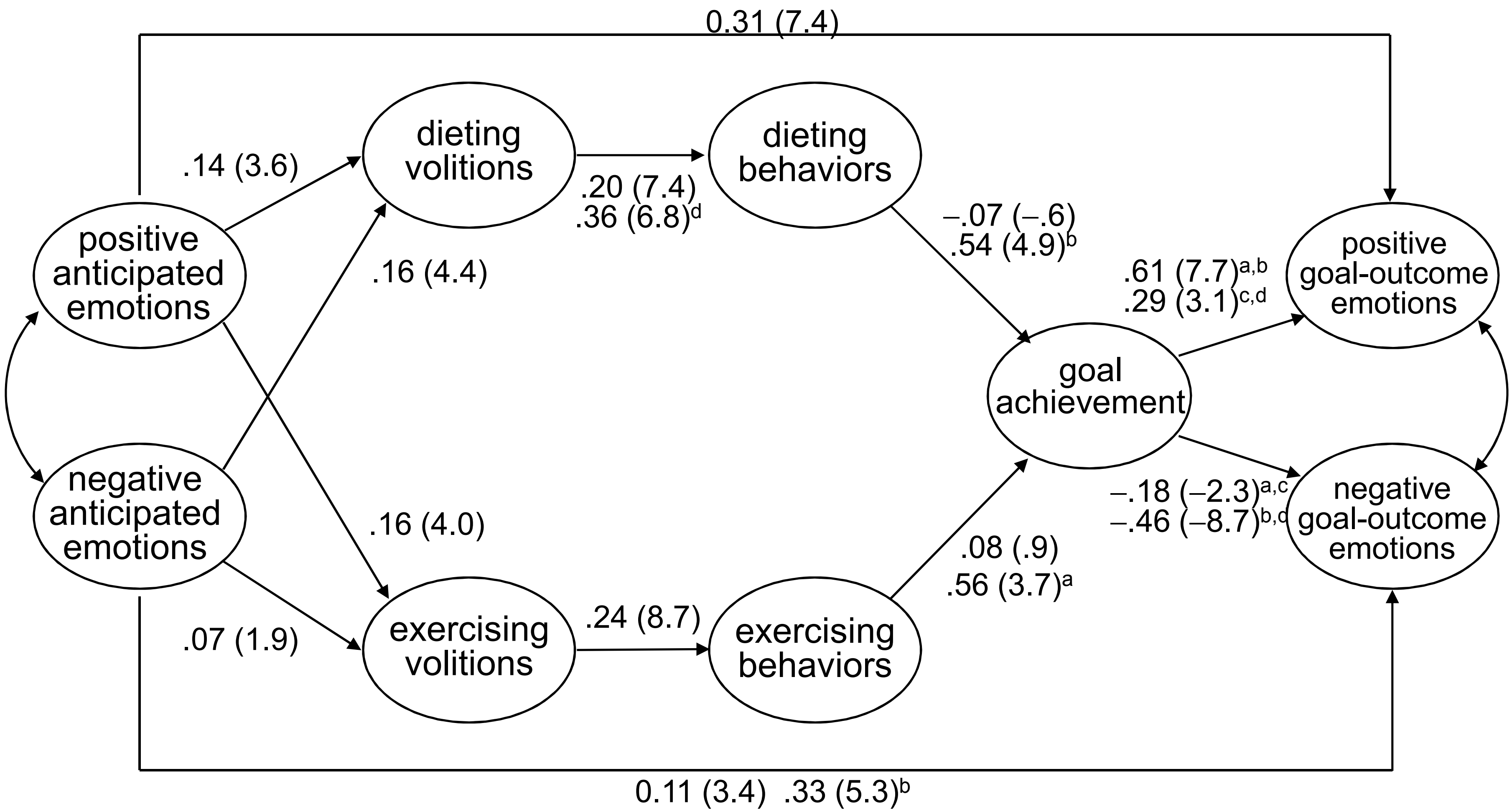
- a mediator Me is a variable that accounts for the relation between a predictor A and a criterion C (i.e., it channels at least some of the total effect of A on C);



- a moderator Mo is a variable that affects the direction and/or strength of the relation between a predictor A and a criterion C;



Mediation and moderation (cont'd)



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$\chi^2(110)=150.51$
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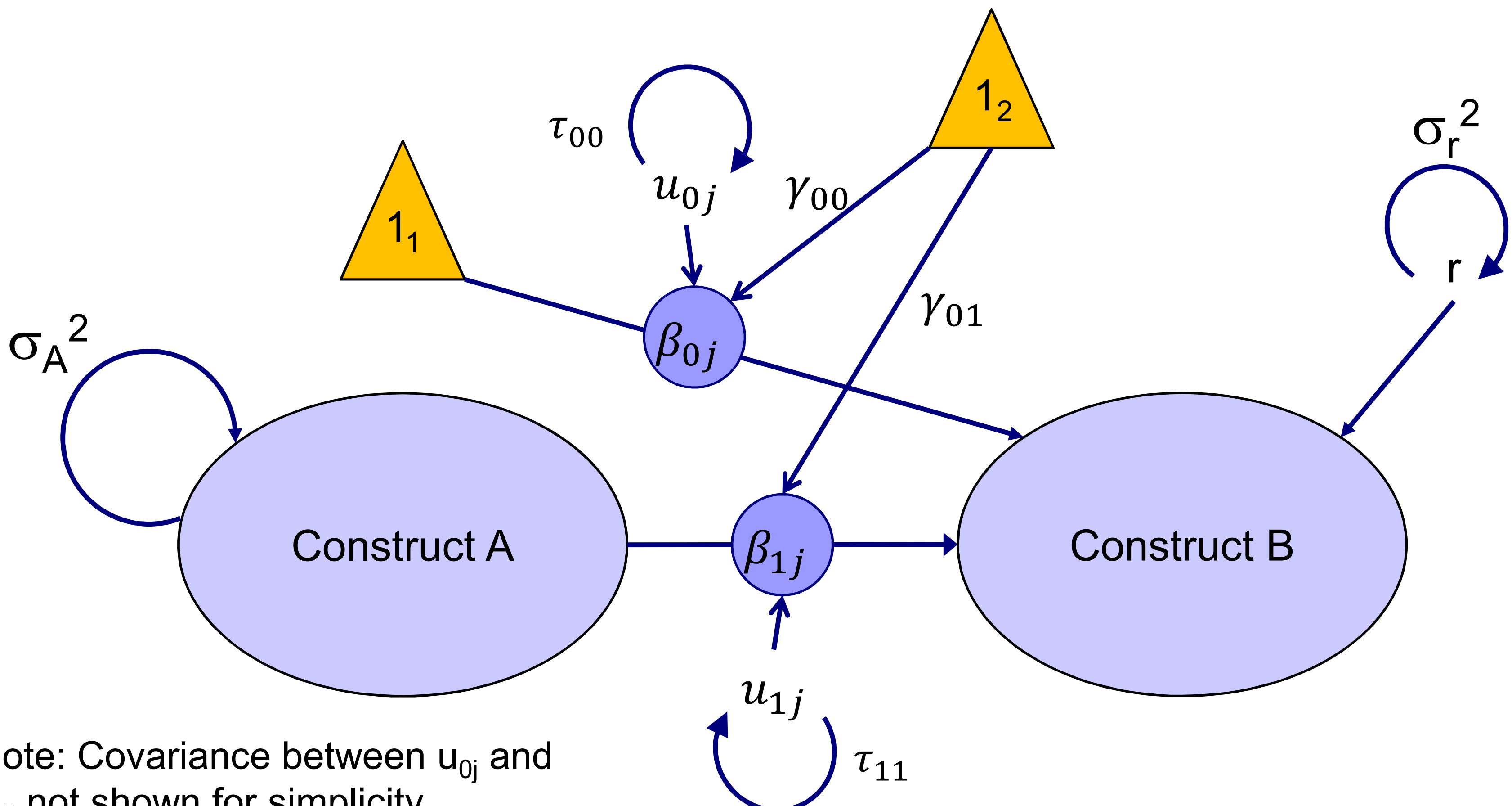
What are the effects of positive anticipated emotions on goal achievement for people who desire to lose weight, and do these effects differ by gender?

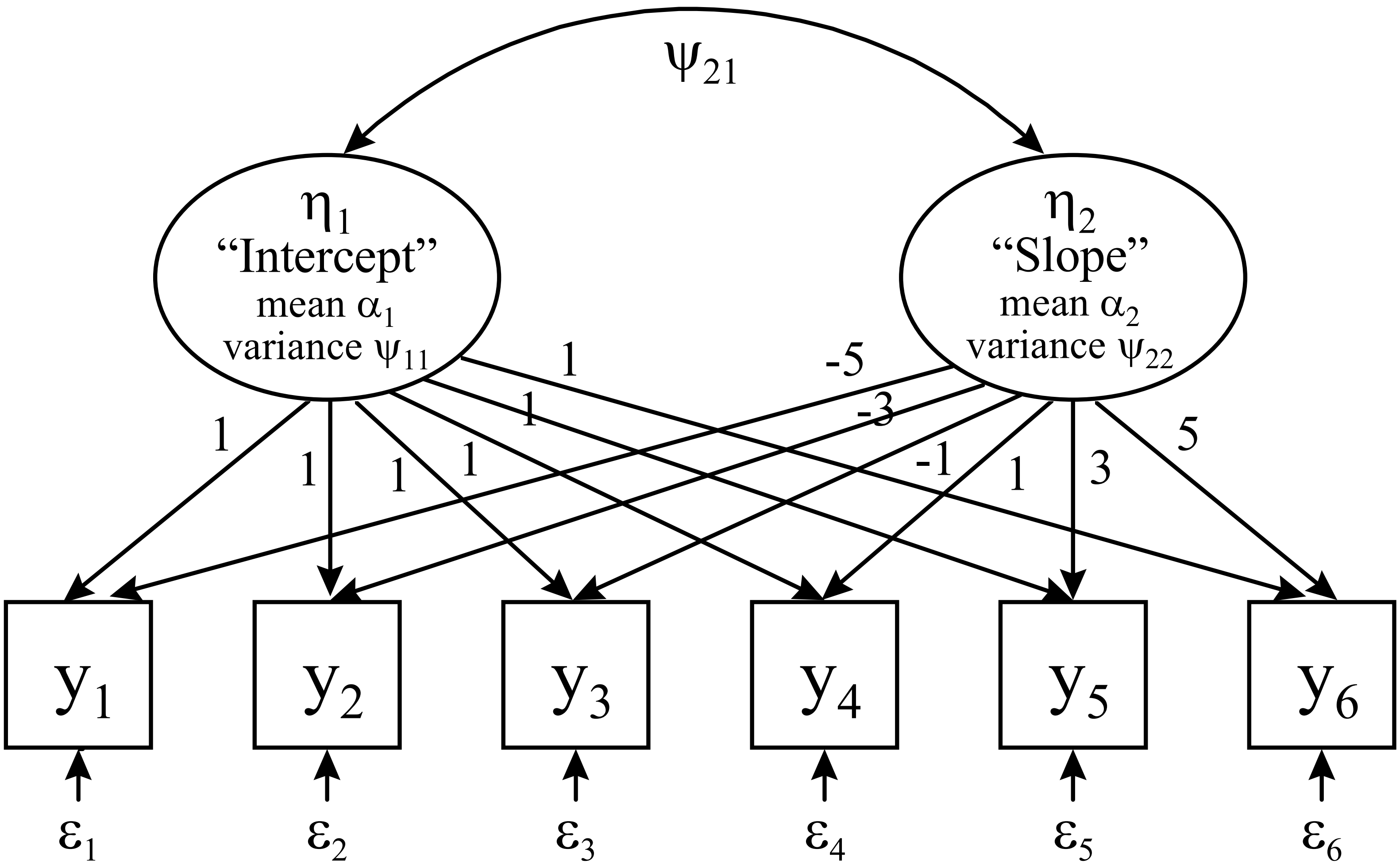
	Direct effect	Indirect effect	Total effect
Males	---	.019*	.019*
via dieting		-.002	
via exercising		.021* s.	
Females	---	.017*	.017*
via dieting		s. .014*	
via exercising		.003	

Hierarchical models

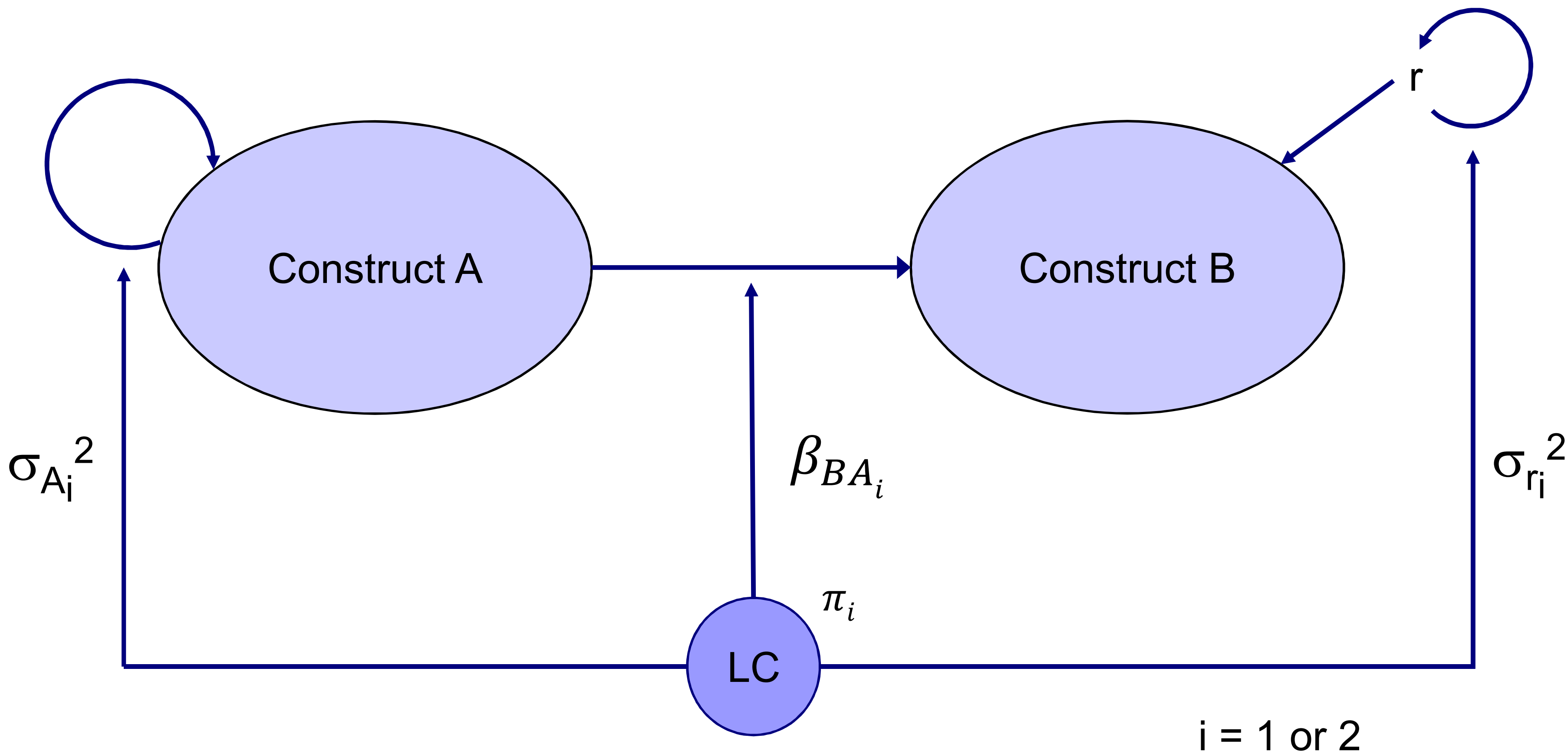


Note: Covariance between u_{0j} and u_{1j} not shown for simplicity.

Latent curve models



Mixture modeling: Unobserved population heterogeneity



Note: The parameters π_i are the mixing probabilities.

Background readings

- Kline, Rex B. (2011), Principles and practice of structural equation modeling, 3rd ed., New York: The Guilford Press.
- Bollen, Kenneth A. (1989), Structural equations with latent variables, New York: Wiley.
- Byrne, Barbara M. (1998), Structural Equation Modeling with LISREL, PRELIS, and SIMPLIS: Basic Concepts, Applications, and Programming, Mahwah, NJ: Erlbaum.

Computer programs for SEM

- LISREL 9.1 (Jöreskog & Sörbom)
  - http://www.ssicentral.com/lisrel/index.html
- Mplus Version 7 (Muthen)
  - http://www.statmodel.com/
- EQS 6.2 (Bentler)
  - http://www.mvsoft.com/eqs60.htm
- PROC CALIS in SAS, AMOS in SPSS, special packages in R, Stata, etc.

Criteria for distinguishing between reflective and formative indicator models

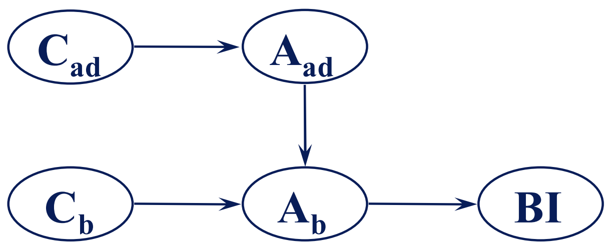
- Are the indicators manifestations of the underlying construct or defining characteristics of it?
- Are the indicators conceptually interchangeable?
- Are the indicators expected to covary?
- Are all of the indicators expected to have the same antecedents and/or consequences?

Based on MacKenzie, Podsakoff and Jarvis, JAP 2005, pp. 710-730.

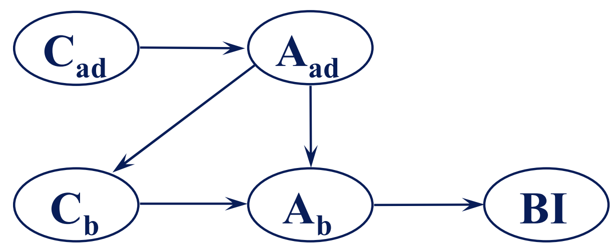
Consumer Behavior
Attitudes

A_{ad} as a mediator of advertising effectiveness: Four structural specifications (MacKenzie et al. 1986)

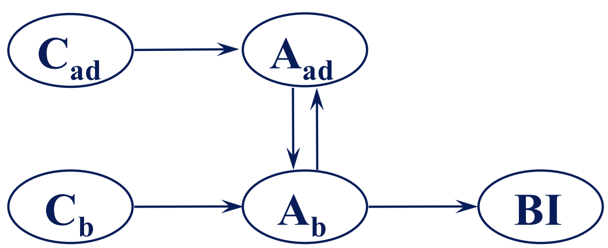
Affect transfer hypothesis



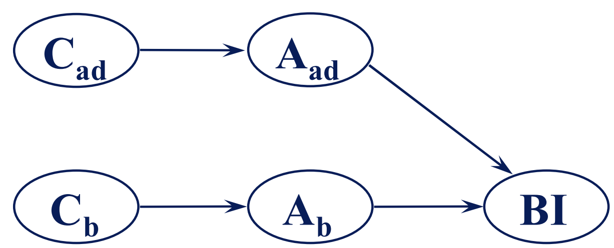
Dual mediation hypothesis



Reciprocal mediation hypothesis



Independent influences hypothesis



Reliability for congeneric measures

- **individual-item reliability** (squared correlation between a construct ξ_j and one of its indicators x_i):

  $$\rho_{ii} = \lambda_{ij}^2 var(\xi_j)/[\ \lambda_{ij}^2\ var(\xi_j) + \theta_{ii}]$$

- **composite reliability** (squared correlation between a construct and an unweighted composite of its indicators $x = x_1 + x_2 + ... + x_K$):

  $$\rho_c = (\Sigma\lambda_{ij})^2\ var(\xi_j)/[\ (\Sigma\lambda_{ij})^2\ var(\xi_j) + \Sigma\theta_{ii}]$$

- **average variance extracted** (proportion of the total variance in all indicators of a construct accounted for by the construct; see Fornell and Larcker 1981):

  $$\rho_{ave} = (\Sigma\lambda_{ij}^2)\ var(\xi_j)/[\ (\Sigma\lambda_{ij}^2)\ var(\xi_j) + \Sigma\theta_{ii}]$$

SIMPLEX specification

```
Title
A general structural equation model (explaining coupon usage)
Observed Variables
id be1 be2 be3 be4 be5 be6 be7 aa1t1 aa2t1 aa3t1 aa4t1 bi1 bi2 bh1
Raw Data from File=d:\m554\eden2\sem.dat
Latent Variables
INCONV REWARDS ENCUMBR AACT BI BH
Sample Size 250
Relationships
be1 = 1*INCONV
be2 = INCONV
be3 = 1*REWARDS
be4 = REWARDS
be5 = 1*ENCUMBR
be6 = ENCUMBR
be7 = ENCUMBR
aa1t1 = 1*AACT
aa2t1 = AACT
aa3t1 = AACT
aa4t1 = AACT
bi1 = 1*BI
bi2 = BI
bh1 = 1*BH
AACT = INCONV REWARDS ENCUMBR
BI = AACT
BH = BI
Set the Error Variance of bh1 to zero
Options sc rs mi wp
Path Diagram
End of Problem
```

Modeling random and systematic measurement error (Baumgartner and Steenkamp 2006)