

# A Supply Chain Tracking Model Using Auto-ID Observations

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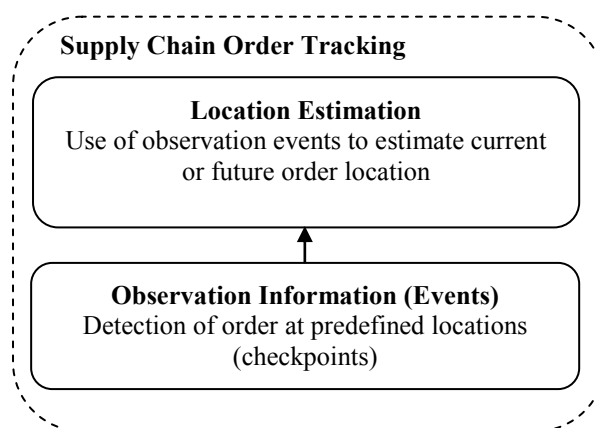
## ABSTRACT

Order location information is undoubtedly one of the most critical pieces of supply chain information. Yet supply chain visibility generally remains a challenge as observations of order progress are often irregular and collected manually. The emergence of Automated Identification (Auto-ID) technologies like Radio Frequency Identification (RFID) is improving the effectiveness of supply chain tracking systems. In this paper we propose a model that describes how Auto-ID observations across a supply chain and historical observation data can be combined to produce an ongoing order location estimation over time. The model is based on probabilistic reasoning principles and the resulting location estimation can be used to support operational decisions as well as to assess the quality and value of tracking information. We provide explicit instructions as to how to use the proposed model and using an illustrative example we demonstrate how the model can produce ongoing location estimates based on RFID read events.

*Keywords: tracking information; model; Auto-ID; order; supply chain; visibility*

## INTRODUCTION

Order location information is the cornerstone for effective decision making in many supply chain operations. Amongst others, ordering and scheduling decisions are directly based on either a current or future estimate of the location of goods moving through the supply chain. This estimate is typically generated based on a series of observations at specific locations across the supply chain (usually called *checkpoints*) as the products move through the chain. This set of observations along with an appropriate model enables estimation of the current or future location of products. We will refer to the overall process of determining the ongoing location of an order in a supply chain as *supply chain order tracking* (Jansen-Vullers, van Dorp, & Beulens, 2003). Figure 1 provides a representation of this definition, in which observation information is used to drive a location estimation process. The final location estimate is used by managers of industrial operations to answer questions like “Where is the order that I am expecting?” or “Will my order arrive by noon tomorrow?”. The answers to these questions are used to make critical decisions regarding inventory management and resource allocation throughout the supply chain.



**Figure 1:** Components of supply chain order

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The emergence of automated identification (Auto-ID) technologies, such as radio frequency identification (RFID) over the last ten years, has created a great potential for the improvement of information quality generated from tracking systems and, as a result, for an improvement of the effectiveness of decisions that use this information. In particular,

- i. RFID technology offers automated object detection enabling the installation of more frequent scan-points throughout the supply chain at a relatively low cost compared, for example, to barcode (Hodges & McFarlane, 2004)
- ii. The automated nature of data capture together with the potential density of data collection points can significantly improve the completeness and the timeliness of the location information delivered.
- iii. Automated network data sharing, for example through the Electronic Product Code (EPC) Network (EPCglobal, 2005), enables supply chain partners to build up the location history of a specific item by retrieving data from across the supply chain using standardized interfaces.

Given the above data characteristics offered by Auto-ID technologies, order location estimation using probabilistic techniques has more chances of delivering realistic estimates than ever before. This paper proposes a model that takes advantage of the potential offered by Auto-ID technologies in order to deliver enhanced order location estimates.

Although Auto-ID technologies are expected to deliver frequent high quality tracking information (Bose & Pal, 2005), the benefits offered need to be carefully balanced with its costs. At the same time the large amount of data that new technologies can generate make it difficult to determine the business value of the applications that use them (Bose & Lam, 2008). As a result, the need to accurately quantify the expected benefits from Auto-ID based tracking systems has raised significant research interest from both academics and practitioners (Whitaker, Mithas, & Krishnan, 2007; Delen, Hardgrave, & Sharda, 2007; Dutta, Lee, & Whang, 2007; Ozelkan & Galambosi, 2008). In order to quantify the impact of these new supply chain tracking systems, subject to different possible system configurations and accuracy levels, it is necessary to formally describe tracking information and the resulting location estimate. Figure 2 describes the rationale and scope of this paper. The output of such a model, concerning the accuracy and timeliness of changes in the location of an order, can then be used to study the impact of tracking information quality on supply chain operations and decisions. In this case, the tracking model can actually form an input into existing value of information calculations like the one proposed by Lawrence (1999). Ultimately, the evaluation of the information impact on the company's operations can provide input to higher level tactical or strategic decisions, for example system configuration or investment decisions.

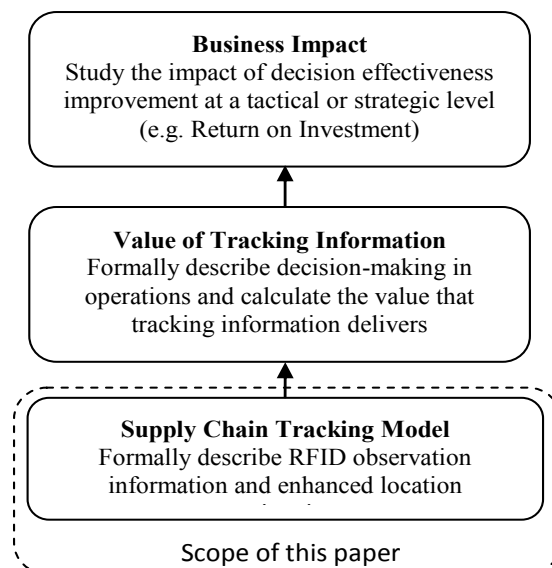


Figure 2: Conceptual representation of the rationale and scope of paper

The aim of this paper is to present an analytic model that delivers enhanced supply chain tracking information based on Auto-ID read events and offers the basis for the objective evaluation of the quality and value of the tracking information. For the development of this model we build on previous aspects of the

model previously presented in Kelepouris, McFarlane, & Parlikard (2007) and Cambridge University, BT Research, & SAP Research (2007). More specifically, the paper's objectives are:

- i. To propose a mathematical model for order location that
  - a. Describes order movement in a linear supply chain
  - b. Incorporates both observation (or tracking) information generated by checkpoints along the supply chain and historical information
  - c. Estimates order location (both current and future), expressed as a location probability distribution based on the above information
- ii. To demonstrate how the proposed model can deliver location estimates based on Auto-ID read events through an example.
- iii. To propose ways in which the model can be used in other research areas of operations management.

This paper is organized as follows: in the following section we provide the background for this study, focusing on principles of probabilistic inference and previous work that demonstrates the criticality of location information in a supply chain and the need for a way to accurately and realistically quantify its value. Then we present the proposed model, describing our modeling approach and the mathematical tools that should be used to produce a location estimate based on historical and observation information. The usage of the model is demonstrated through an example and we conclude this paper in the final section.

## BACKGROUND

Previous work on supply tracking applications and existing industrial practices have focused on the systems' architecture and the identification technology used rather than on modeling the generated tracking information in order to demonstrate the business benefits. For example, Kärkkäinen, Ala-Risku, and Främling (2004) suggest an architecture that enables efficient supply chain tracking. Woo, Choi, Kwak, and Kim (2009) present a tracking architecture for products enhanced with sensors which is proposed to be useful for the detection of lost objects and constraint violation. Delen et al. (2007) present a case study in which RFID observation data at a number of checkpoints across a supply chain is used to produce metrics that reflect variability of transit times throughout the chain, which in turn assist decision making. The metrics are based on the observation timestamps and no further reasoning takes place to produce a location estimate of objects, therefore limiting tracking information to the observation events. Jansen-Vullers et al. (2003) suggest a reference model to describe, among others, tracking information, although the process of producing a location estimate is also out of their scope. The above are representative studies of existing industrial practices which demonstrate that tracking information is currently limited purely to observation events, without any subsequent reasoning. Kelepouris, Harrison and McFarlane (2011) suggest an algorithm for the location estimation of an order in a supply chain network, however, without building an overall tracking model for the quantification of the value of this information. Given the fact that a tracking system is the link between the actual world and the manager's perception of it (Kärkkäinen et al., 2004), it becomes clear that the quality of the observation data and the final location estimate are critical for supporting the manager's decision. However, a formal way to analytically describe tracking information and enable probabilistic order location estimation in the supply chain is missing.

Kelepouris, Da Silva, and McFarlane (2006) suggest that there are four main factors that affect the quality of supply chain tracking information: i) processing delays during order identification, ii) accuracy of identification technology, iii) configuration of sensor devices across the supply chain and iv) accuracy of aggregation information when objects are tracked in merged shipments. The emergence of new identification technologies, and the evolution of satellite-based location systems have created a great potential to overcome problems regarding the above factors that affect the quality of tracking information. New technologies promise to deliver much better information at affordable cost. This is one of the reasons why RFID has been recognised as a promising technology that can eventually replace barcode, especially in supply chain operations (Soon & Gutiérrez, 2009). However, the accurate quantification of the expected benefits from the use of these technologies remains a challenging task.

Many studies and industrial reports provide estimates of the value of these technologies; however, they generally provide only limited evidence about the robustness of the quantification method used. For example, as Dutta et al. (2007) note, in order to produce an accurate and realistic valuation of RFID technology one should model in detail the ground-level operational characteristics of the processes involved and not make

unsupported assumptions on how the technology might affect the process. The academic literature includes a number of studies that have calculated the value that Auto-ID technologies deliver through improved supply chain visibility, by modeling operations in a detailed level. In particular, Gaukler, Özer, and Hausman (2004) calculate the value of RFID for enhanced order progress visibility, enabling the optimization of the order policy. Al Kattan and Al Khudairi (2010) investigate the benefits that different Auto-ID technologies can bring to inventory control systems and compare their value in scenarios with different types of demand. Ozelkan and Galambosi (2008) propose a modeling framework to quantify the benefits and risks of RFID deployments from a financial perspective, studying factors such as break-even sales volumes, unit profits and tag prices. These studies make a significant contribution to tracking but what is still needed is an analytic model to produce an order location estimate over time using observation events.

In order to accurately assess the value of tracking information, the observation events and the resulting ongoing location estimate of an order's location through the supply chain should be explicitly modeled when studying their impact on business operations for the following reasons:

- i. The tracking information generated by the tracking system and the resulting location estimate can be different from the actual location of an order in the supply chain. This intrinsic deviation should be reflected by the modeling approach.
- ii. Identification technologies are not perfect; therefore it is likely that there will be some missed scans at checkpoints along the supply chain which should be taken into account. This will affect the accuracy of tracking information and therefore its value.
- iii. The configuration of the tracking system, for example the number and the location of sensing devices, affects the tracking information delivered by the system.

The model proposed in this paper aims to provide a ground-level description of supply chain tracking information and the way this is generated, without making any assumptions about the underlying identification technology or the processes that generate the information. We believe that this enables us to assess the value of tracking information in a realistic manner.

In order to develop a model for supply chain tracking information, we use principles from existing object tracking techniques. Russell and Norvig (2003) provide a very good overview of the basic principles of probabilistic reasoning, which we will use as the basis for order tracking using observation information. They show how state and evidence variables and the relationship between them should be modeled in order to produce a state estimation based on values of the evidence variables. They demonstrate the use of these principles in the context of Kalman filters and Hidden Markov Models. Based on the same probabilistic principles, Liao, Fox, Hightower, Kautz, and Schulz (2003) propose a method for indoor tracking based on sparse observation data. They treat the location space in a discrete manner and they use sampling techniques to reduce computational complexity. The work of McKenna and Nait-Charif (2007) is a similar example that demonstrates the use of a particle filtering technique for motion tracking. We adopt the basic modeling principles from the above studies and we adapt them to the context of supply chain tracking.

To summarize, existing industrial approaches and previous academic work show that there is a need for a model that formally describes supply chain tracking information and enables enhanced location estimation, which will provide the basis for the accurate quantification of benefits that this can bring to a company. In this paper we aim to address this gap by proposing a model that describes the way that Auto-ID tracking information is generated and uses probabilistic reasoning techniques to produce a location estimate for an order over time. The model presented in this paper is agnostic with respect to the data capture technology, so that one can study the effect of different identification technologies on the quality of tracking information and supply chain visibility.

## **THE TRACKING MODEL**

In this section we propose a model for describing order location estimation based on:

- i. historical order shipment data
- ii. sensor checkpoint data
- iii. a simple flow model for order movement

Our approach is based on Bayesian reasoning and follows the style of approach of Russell and Norvig (2003). The model comprises of the following key components:

- i. A conditional distribution that describes the relationship between consecutive order locations, called the transition model
- ii. A conditional distribution that describes the relationship between the tracking data signals and the actual order location, i.e. how the tracking system captures order location. This is called the sensor model
- iii. The inference calculations that combine the above models with actual observation data in order to produce an estimate for an order's location.

We begin by introducing the notation used and in the following subsections we analyze the above components first for general assumptions about the distributions and then for the special case where the distributions are assumed to be normal.

## Notation

In the developments that follow, we will use the following notation for the model variables:

$t$  : index of time period  
 $S = [0, l]$  : possible order location space  
 $X_t$  : location variable of order at time  $t$   
 $x_t$  : value of order location at time  $t$   
 $x_{1:t}$  : sequence of order location values  $x_t$  for time steps 1 to  $t$   
 $N_c$  : number of checkpoints along the supply chain  
 $c_i$  :  $i^{\text{th}}$  checkpoint along the supply chain,  $i = 1, 2, \dots, N_c$   
 $Y^{c_i}$  : information signal variable resulting from checkpoint  $c_i$   
 $y_t^{c_i}$  : information signal value at time  $t$  resulting from checkpoint  $c_i$   
 $y_{1:t}^{c_i}$  : sequence of information signal values  $y_t^{c_i}$  for time steps 1 to  $t$   
 $Z$  : state space  
 $m$  : number of states in  $S$   
 $z^i$  :  $i^{\text{th}}$  state in  $S$ ,  $i = 1, 2, \dots, m$   
 $\bar{z}^i$  : lower limit of state  $z^i$   
 $\mu_t$  : mean of a normal distribution at time  $t$   
 $\sigma_t^2$  : variance of a normal distribution at time  $t$   
 $\sigma^2$  : variance of a normal transition model  
 $\alpha$  : normalizing constant used to make probabilities sum up to 1  
 $f(t)$  : a function representing the expected movement of the order over time

## Model Development

### *Expected Order Location Evolution: The Transition Model*

Let  $X_t$  denote the location of an order that is moving across a linear supply chain. If  $l$  denotes the chain's "length", then  $X_t \in S = [0, l]$ . Capital letters denote variables and lower case letters refer to their instances. At any time  $t$  the possible location of the order can be described with a probability distribution over  $X_t$ , where we use the abbreviation  $P(X_t = x_t) = P(X_t)$ . Under the Markov assumption (Rabiner, 1989), the evolution of the order's location can be described as a first order Markov process defined by

$$P(X_{t+1} | X_{0:t}) = P(X_{t+1} | X_t) \quad (1)$$

where  $X_{1:t}$  denotes the sequence of order location for time steps 1 to  $t$ . The relation between consecutive instances of  $X_t$ , described in (1), is called a *transition model* (Russell & Norvig, 2003). Given (1) and the

location probability distribution at time  $t$  **Error! Reference source not found.**,  $P(X_t)$ , we can compute the one step prior location estimation  $P(X_{t+1})$

$$P(X_{t+1}) = \int_{-\infty}^{+\infty} P(X_{t+1} | x_t) P(x_t) dx_t \quad (2)$$

Using (2) repeatedly we can also estimate the order's location distribution for  $n$  time steps ahead, starting from a prior distribution  $P(X_0)$ .

### ***Tracking Information: The Sensor Model***

Let  $c_1, c_2, \dots, c_{N_c} \in [0, l]$  denote checkpoint locations in the supply chain. Also, for notation simplicity, let the checkpoint index indicate their sequence in the supply chain, i.e. if  $j > i$ ,  $c_j$  is reached after  $c_i$  as the order moves across the supply chain. At each checkpoint, there is a sensor that detects and records the presence of the order at that particular location. We define the following variables referring to checkpoint  $c_i$ :

$$E_t^{c_i} = \begin{cases} 0 & \text{object is not detected at } c_i \text{ at time } t \\ 1 & \text{object is detected at } c_i \text{ at time } t \end{cases} \quad (3)$$

and

$$H_t^{c_i} = \begin{cases} 0 & \text{at time } t \text{ object has not yet been detected at } c_i \\ 1 & \text{object has previously been detected at } c_i \text{ at time } t_h: t_h \leq t \end{cases} \quad (4)$$

We define a *tracking signal*  $Y_t^{c_i}$  as the vector consisting of the combination of values of  $E_t^{c_i}$  and  $H_t^{c_i}$ ,  $Y_t^{c_i} = (E_t^{c_i}, H_t^{c_i})$ . It follows from (3) and (4) that the possible values for  $Y_t^{c_i}$  are (0,0), (0,1) and (1,1). The way that a tracking signal responds to the movement of the order across the supply chain, that is the value it will take depending on the order's location, is described by the conditional distribution  $P(Y_t^{c_i} = y_t^{c_i} | X_t = x_t)$ , which is called *sensor model* or *observation model* (Russell & Norvig, 2003). We assume that the value of the tracking signal at any time depends only on the order's location at that time, that is:

$$P(Y_t^{c_i} = y_t^{c_i} | X_{0:t} = x_{0:t}, Y_{0:t-1}^{c_i} = y_{0:t-1}^{c_i}) = P(Y_t^{c_i} = y_t^{c_i} | X_t = x_t) \quad (5)$$

### ***Inference Calculation: Location Estimation Using Observation Information***

Given the values of tracking signal  $y_{1:t+1}^{c_i}$  for time steps  $1, 2, \dots, t+1$ , we can revise our estimation about the order's location according to the following:

$$\begin{aligned} P(X_{t+1} | y_{1:t+1}^{c_i}) &= P(X_{t+1} | y_{1:t}^{c_i}, y_{t+1}^{c_i}) \\ &= \alpha P(y_{t+1}^{c_i} | X_{t+1}, y_{1:t}^{c_i}) P(X_{t+1} | y_{1:t}^{c_i}) \text{ (Bayes' rule)} \\ &= \alpha P(y_{t+1}^{c_i} | X_{t+1}) P(X_{t+1} | y_{1:t}^{c_i}) \text{ (Markov property of evidence)} \end{aligned} \quad (6)$$

We will use  $\alpha$  as normalizing constant to make probabilities sum up to one. The reader is referred to Cambridge University et al. (2007) for a numerical example of the use of the normalising constant (p.31-32). The second term of (6) is the one step prediction, described in (2). Replacing this in (6), we get the order's location estimation, given the tracking signal values up to time  $t+1$ , which is described by the following posterior location probability distribution

$$P(X_{t+1} | y_{1:t+1}^{c_i}) = \alpha P(y_{t+1}^{c_i} | X_{t+1}) \int_{-\infty}^{+\infty} P(X_{t+1} | x_t) P(x_t | y_{1:t}^{c_i}) dx_t \quad (7)$$

In simple words, equation (7) defines that our estimation of the order's location at time  $t+1$  given the tracking signal values  $y_{1:t+1}^{c_i}$  will result from the one step prediction of the order's location based on the previous time step estimation (expressed by the integral in (7)) and then revised based on the tracking signal received at time  $t+1$ . Using this equation and given a set of signal values, we can estimate the order's location, starting from a prior location probability distribution at time 0,  $P(X_0)$ .

### Location Space Partitioning

Equation (7) defines the posterior probability distribution of the order's location  $X_{t+1}$ . However, in some cases, rather than requiring a continuous probability distribution over the actual location, it is more relevant to simply have a distribution over different *sets* of locations, where each set relates to different states of the order's evolution. The partitioning of the location space  $S$  into a state space in this way can be done according to any rules imposed by the operational or decision making context in which tracking information is used. For example we might be interested in whether an order is currently in a warehouse, in a dispatching yard or already on its way to a retailer. Specific locations within each of these three points of interest define sets of specific location values from our location space  $S$ . In accordance with Marschak and Radner (1972), we define a *partition* of  $S$  as a collection of states  $z^1, z^2, \dots, z^m$  such that

- i. Every order location  $x$  is in some state  $z^i$
- ii. No order location  $x$  is in two different states  $z^i$  and  $z^k$

Or, equivalently, for any  $x_i \in S$  there is one  $z^j \in Z$  such that

$$P(z^j | x_i) = 1, z^j \in Z \text{ and } P(z^k | x_i) = 0, z^k \in Z : z^k \neq z^j \quad (8)$$

Further, for our linear system, in which we assume states  $\bar{z}^i, \bar{z}^{i+1}$  are adjacent intervals of the location space, we define  $\bar{z}^i, \bar{z}^{i+1} \in [0, l]$ , to be the lower and upper limits of locations in state  $\bar{z}^i$ .

### Posterior State Probability Distribution

Using the posterior probability distribution over the order location  $P(X_t | y_{1:t}^{c_i})$  we can compute the posterior distribution over a state  $z^j$  at time  $t$  given by  $P(Z_t^j | y_{1:t}^{c_i})$  as

$$P(Z_t^j | y_{1:t}^{c_i}) = \int_{x_t \in z^j} P(x_t | y_{1:t}^{c_i}) dx_t \quad (9)$$

or in the case that  $z^j$  is a continuous interval of the location space, the posterior over the state will be given by

$$P(Z_t^j | y_{1:t}^{c_i}) = \int_{\bar{z}^j}^{\bar{z}^{j+1}} P(x_t | y_{1:t}^{c_i}) dx_t \quad (10)$$

The posterior distribution over the order location for  $n$  time steps ahead  $P(X_{t+n} | y_{1:t}^{c_i})$  (for which no tracking signals are available after time  $t$ ) can be computed by starting from the posterior distribution at time  $t$  (7) and applying the step prediction (2) for  $n$  steps. This future posterior distribution can be used to estimate

the distribution over the future state of an order. For example, if the supply chain is partitioned into two location states  $z^1 = [0, \bar{z}^2]$ ,  $z^2 = [\bar{z}^2, l]$ , then the probability that the order is in state  $z^2$  at time  $t+n$  will be given by

$$P(Z_{t+n}^2 | y_{1:t}^{c_i}) = \int_{\bar{z}^2}^l P(x_{t+n} | y_{1:t}^{c_i}) dx_{t+n} \quad (11)$$

Table 1 summarizes the probability distributions that need to be populated as well as the necessary calculations in order to produce a location or state estimation. In a later section we will demonstrate how to populate the distributions and how to use the proposed formulas.

### Special Case: Normal Distribution

We now reconsider the results in (2) for the special case where the prior distribution  $P(x_0)$  and the transition model  $P(x_{t+1} | x_t)$  are normal. We focus on the normal distribution for two reasons: i) it represents order location uncertainty in a realistic way and ii) it offers some convenient mathematical properties. However, we need to stress on the fact that the model is not limited to the normal case, but rather that this case has been selected for presentation reasons.

Let the prior distribution be  $P(x_0) = \alpha e^{-\frac{1}{2} \left( \frac{(x_0 - \mu_0)^2}{\sigma_0^2} \right)}$  and in the general case the location probability distribution at time  $t$  be

$$P(x_t) = \alpha e^{-\frac{1}{2} \left( \frac{(x_t - \mu_t)^2}{\sigma_t^2} \right)} \quad (12)$$

where  $\mu_t$  is the mean and  $\sigma_t^2$  is the distribution variance at time  $t$ . The normal transition model can be expressed as

Table 1: Summary of required distributions and calculations for using the model

Step	Formula
Definition of prior distribution, transition model and sensor model	Define and populate $P(X_0)$ , $P(X_{t+1}   X_t)$ and $P(Y_t   X_t)$
One-step location prediction	$P(X_{t+1}) = \int_{-\infty}^{+\infty} P(X_{t+1}   x_t) P(x_t) dx_t$
Posterior location probability distribution	$P(X_{t+1}   y_{1:t+1}^{c_i}) = \alpha P(y_{t+1}^{c_i}   X_{t+1}) \int_{-\infty}^{+\infty} P(X_{t+1}   x_t) P(x_t   y_{1:t}^{c_i}) dx_t$
Definition of state partition of the location space	Define $Z$
Posterior state probability distribution	$P(Z_t^j   y_{1:t}^{c_i}) = \int_{x_t \in z^j} P(x_t   y_{1:t}^{c_i}) dx_t$

$$P(x_{t+1} | x_t) = \alpha e^{-\frac{1}{2} \left( \frac{(x_{t+1} - (x_t + f(t)))^2}{\sigma^2} \right)} \quad (12)$$

Equation (13) defines that the transition model adds a normal perturbation of variance  $\sigma^2$  and shifts the mean of the distribution according to the function  $f(t)$ . The one step predicted distribution at time  $t+1$  can be computed according to (2) and results in



$$\begin{aligned}
P(x_{t+1}) &= \int_{-\infty}^{+\infty} P(x_{t+1} | x_t) P(x_t) dx_t = \\
&\alpha \int_{-\infty}^{+\infty} e^{-\frac{1}{2} \left( \frac{(x_{t+1} - (x_t + f(t)))^2}{\sigma^2} \right)} e^{-\frac{1}{2} \left( \frac{(x_t - \mu_t)^2}{\sigma_t^2} \right)} dx_t \quad (14) \\
&= \alpha e^{-\frac{1}{2} \left( \frac{(x_{t+1} - (\mu_t + f(t)))^2}{\sigma_t^2 + \sigma^2} \right)}
\end{aligned}$$

which indicates that when a normal transition model is applied to a normal distribution, the resulting distribution is also normal. A complete proof is provided in Kelepouris (2008). Equation (14) shows that the variance of the predicted distribution  $P(x_{t+1})$  will result from the original variance of the previous step, increased by the variance of the transition model ( $\sigma_{t+1}^2 = \sigma_t^2 + \sigma^2$ ). The predicted distribution's mean is a shift of the original mean  $\mu_t$  by  $f(t)$ ,  $\mu_{t+1} = \mu_t + f(t)$ .

The definition of the sensor model in the normal case depends on the way one interprets each information signal with regard to the possible order location that may have generated it. We present some options which preserve the convenient properties of the normal case. For each order location  $x_k$  we introduce the factors  $\beta_k$  and  $\gamma_k$  that indicate the conditional probability of receiving specific signal values when the order is at that location, as described in Table 2.

When  $Y_t^{c_i} = (0,0)$  one could assume that the possible order locations that could have generated this value are the entire location space  $S$  excluding  $c_i$ , expressed by (15), or in a different case all locations earlier than  $c_i$ ,  $x_t < c_i$ , expressed by (16). For  $Y_t^{c_i} = (1,1)$ , it is reasonable to assume that the order is at the checkpoint location as expressed by (17). Similarly to the (0,0) signal, when  $Y_t^{c_i} = (0,1)$  we can assume that the value was generated by any location apart from  $c_i$ , expressed mathematically by (18), or it was generated by any location further in the supply chain than  $c_i$ , expressed by (19).

The above options of the sensor model ensure that the resulting posterior distribution at each time step is still normal (or it is very well approximated by one, in the case where one of the distribution's tails is truncated due to the sensor model). It should be noted that if each location  $x_k$  is mapped to only one signal value, then all factors  $\beta_k, \gamma_k$  should be set equal to 1. It is very important that factors  $\beta_k$  and  $\gamma_k$  are chosen in way that the model has statistical consistency, defined by

$$\sum_{y^{c_i}} P(y^{c_i} | x_t) = 1 \quad (20)$$

Equations (15) - (19) are an example of the possible configurations of the sensor model. One could appropriately modify them to better reflect the real operation of the tracking system. However, the effect on the posterior distribution calculation should be carefully taken into account, as some convenient properties of the model might be lost. The example in subsection "Example" demonstrates the use of this model variation and the resulting posterior distributions for different versions of the sensor model.

Table 2: Sensor model options for the normal version of the supply chain tracking model

Signal Value	Signal Meaning	Sensor Model		Sensor Model Meaning
$Y_t^{c_i} = (0, 0)$	By time $t$ , order has not yet been observed at checkpoint $c_i$	Option 1:	$P(Y^{c_i} = (0, 0)   x_k) = \beta_k, \forall x_k \in [0, l] - \{c_i\}$ and $P(Y^{c_i} = (0, 0)   x_k) = 0, x_k = c_i$ (15)	The order can be anywhere apart from the checkpoint location $c_i$
		Option 2:	$P(Y^{c_i} = (0, 0)   x_k) = \beta_k$ when $x_k < c_i$ and $P(Y^{c_i} = (0, 0)   x_k) = 0$ when $x_k \geq c_i$ (16)	Order can be in any location $x_k$ before the checkpoint location $c_i$ , $x_k < c_i$
$Y_t^{c_i} = (1, 1)$	Order is being observed at checkpoint $c_i$ at time $t$	$P(Y^{c_i} = (1, 1)   x_k) = 1$ when $x_k = c_i$ and $P(Y^{c_i} = (1, 1)   x_k) = 0$ when $x_k \neq c_i$ (17)		Order can only be at the checkpoint location $c_i$
$Y_t^{c_i} = (0, 1)$	Order was observed at checkpoint $c_i$ at a time earlier than $t$	Option 1:	$P(Y^{c_i} = (0, 1)   x_k) = \gamma_k, \forall x_k \in [0, l] - \{c_i\}$ and $P(Y^{c_i} = (0, 1)   x_k) = 0, x_k = c_i$ (18)	The order can be anywhere apart from the checkpoint location $c_i$
		Option 2:	$P(Y^{c_i} = (0, 1)   x_k) = \gamma_k$ when $x_k \geq c_i$ and $P(Y^{c_i} = (0, 1)   x_k) = 0$ when $x_k < c_i$ (19)	Order can be in any location $x_k$ after the checkpoint location $c_i$ , $x_k \geq c_i$

### Summary of Tracking Model

In this section we have defined a generic model for describing tracking information and calculating location estimation based on tracking signals. We have also presented the special case in which the prior distribution and the transition model are normal. In the next section we demonstrate the way that the model can be used in order to produce a location estimation given a set of tracking signals.

### USING THE SUPPLY CHAIN TRACKING MODEL

As an order moves across a supply chain, observation events at checkpoints along the chain record its location at the time of observation. However, there is no direct information about the order's location between observations. As a consequence, further reasoning is necessary in order to produce an estimate for the order's location at any time while making use of these observation events. In this section we describe the step-by-step process that one should follow in order to use the model and calculate a posterior distribution over time based on sensor information. We focus on the normal case, which is convenient for demonstrating the use of the model and interpreting its output through graphical representation. We also analyze in detail how the model parameters should be determined, as this is the most critical task for the model to deliver realistic output.

## How to Calculate a Posterior Location Probability Distribution

The steps that should be taken in order to use the model can be split into three stages:

1. Static information definition, where the analyst should define the parameters that define the operational context in which the model will be used, such as the number of checkpoints, the layout of the supply chain.
2. Historical information collection, where past observation information should be collected in order to extract knowledge about the likely movements of current orders.
3. Model execution (location prediction), during which the mathematical equations that were presented in the previous section (Table 1) should be applied to produce the desired output.

### *Step 1: Static Information Definition*

In the first step the user should define the following static parameters that define the operational context in which the model will be used:

- i. The units in which  $X$  shall be measured and the location space  $S = [0, l]$  in which the order location is defined.
- ii. The location(s)  $c_i$  of checkpoints in  $S$ .
- iii. The limits  $\bar{z}^i$  of states along the supply chain. In the case that the location space is treated as discrete, each state  $z^i$  can be defined as an arbitrary collection of locations  $x$  rather than an interval  $[\bar{z}^i, \bar{z}^{i+1}] \subseteq [0, l]$ .

This information will be used in the model execution stage.

### *Step 2: Historical Information Collection*

In this step the user collects historical location information, i.e. past information about location of orders in the supply chain, which shall be used in the next stage to populate the dynamic model parameters such as the transition and sensor model. Historical information can either be complete or sparse. Complete historical location information provides the location of orders at any point in time as they moved across the supply chain in the past. Sparse historical location information refers to data that indicate the time that orders reached specific locations (usually checkpoints) across the supply chain. In a real case, the model user is more likely to have access to sparse information. Subsection "Populating the model parameters" describes how this type of historical information can be used to determine the model parameters such as the transition model.

### *Step 3: Model Execution*

In the final stage, the user uses the collected information and can calculate the posterior location probability distribution over time given a set of observations, by taking the following steps as described in Table 1:

- i. Populating the dynamic model parameters: the prior distribution, the transition model and the sensor model.
- ii. Calculation of the posterior location probability distribution: Given a set of observation times at the checkpoints, starting from the prior distribution  $P(X_0)$  at  $t=0$ , for each time  $t$  apply the one step prediction (2) and calculate the revised posterior location probability distribution according to (7). This will create a posterior location probability distribution for each time step.
- iii. Calculation of posterior probability distribution over  $Z$ : Based on the posterior calculated in the previous step, calculate the posterior over the partition  $Z$  according to (10).

The population of the dynamic model parameters is probably the most critical step with regard to the accuracy of the model and the least straightforward of the ones proposed. In the next subsection we analyze how the dynamic model properties should be populated using either expert knowledge or preferably, historic location information.

## Populating the Model Parameters

The extent to which the posterior distribution reflects the actual order movement heavily depends on the right choice of the model parameters, namely the prior distribution  $P(X_0)$ , the parameters of the transition model and the parameters of the sensor model. Table 3 summarizes the parameters that need to be determined in order to use the normal version of the supply chain tracking model.

If no historical data are available regarding the detection of orders at the checkpoints, then the parameters must be estimated using expert knowledge. There are several techniques that suggest ways for eliciting subjective probabilities from expert knowledge, which can be used in this context. The studies of Chesley (1978) and Ludke, Stauss, and Gustafson (1977) provide a comparison between some of the methods. Moreover, some factors that should be taken into account when following this approach, such as bias, are analyzed in the following studies (Bunn, 1975; Bunn 1979; Spetzler & Stael Von Holstein, 1975; Winkler, 1967; Wright & Ayton, 1987). After being initialized using expert knowledge, the model

Table 3: Dynamic model parameters that need to be determined for using the model

Distribution	Parameters that need to be determined
Prior distribution, $P(X_0)$	$\mu_0$ : mean of the prior normal distribution
	$\sigma_0^2$ : variance of the prior normal distribution
Transition model, $P(X_{t+1}   X_t)$	$f(t)$ : mean shift function of the transition model
	$\sigma^2$ : variance of the transition model distribution
Sensor model, $P(Y_t   X_t)$	Definition of factors $\beta_k$ and $\gamma_k$ for each location $x_k$
	Selection between the options of Table 2

parameters can be fine tuned once observation data become available.

Using historical observation data, the model parameters can be estimated much more accurately. In the normal case the data can be used to estimate the parameters of the transition model  $f(t)$  and  $\sigma^2$ . There is no need to use the data for estimating the prior or the sensor model, since the prior distribution will probably span around the beginning of the location space (where the order begin from) and the sensor model can have the form of equations (15) - (19). In the unlikely case that the data provide the exact location of each order at any time, the calculation of the required parameters is straightforward, since the data would create a normal distribution for each time step. In that case, finding the values of  $f(t)$  and  $\sigma^2$  that best fit each normal would be a trivial task.

Rather than providing the exact location at any time, the observation data will probably provide detection events at the checkpoints along the supply chain together with the timestamp for each detection. In this case, the parameters in question can be estimated using the following approximation, applied for each pair of consecutive checkpoints  $c_i$  and  $c_{i+1}$ : the observation data will provide detection timestamps at the two checkpoints for a number of orders that moved between them. Normalizing the data such that the detection time at  $c_i$  for all orders is  $t=0$ , we will have a distribution of the time that the orders arrived at  $c_{i+1}$ . Assuming that an order started from  $c_i$  at  $t=0$  and  $f(t)=k$ , its location probability distribution at any time will be

$$P(x_t) = \alpha e^{-\frac{1}{2} \left( \frac{(x_t - (\mu_{t-1} + f(t)))^2}{\sigma_{t-1}^2 + \sigma^2} \right)} \quad (21)$$

Since  $f(t)=k$  the mean at time  $t$  will be  $\mu_t = \mu_0 + kt = c_i + kt$ , therefore

$$P(x_t) = \alpha e^{-\frac{1}{2} \left( \frac{(x_t - (c_i + kt))^2}{\sigma_{t-1}^2 + \sigma^2} \right)} \quad (22)$$

Rewriting (22) as a distribution of the time  $t$  that orders arrive at checkpoint  $c_{i+1}$  we have

$$P(t; x = c_{i+1}) = \alpha e^{-\frac{1}{2} \left( \frac{(c_{i+1} - (c_i + kt))^2}{\sigma_{i-1}^2 + \sigma^2} \right)} = \alpha e^{-\frac{1}{2} \left( \frac{\left[ t - \left( \frac{c_{i+1} - c_i}{k} \right) \right]^2}{\frac{\sigma_{i-1}^2 + \sigma^2}{k^2}} \right)} \quad (23)$$

The mean and variance of the distribution in (23) can be derived from the observation data. Let these be  $\mu_d$  and  $\sigma_d^2$  respectively. We can then compute  $k$ , which essentially represents the average speed between  $c_i$  and  $c_{i+1}$

$$k = \frac{c_{i+1} - c_i}{\mu_d} \quad (24)$$

Assuming that on average an order needs  $\mu_d$  time steps to go from  $c_i$  to  $c_{i+1}$ , according to the transition model (14) the variance of the distribution at time  $t$  will be  $\sigma_d^2 = \sigma_0^2 + \mu_d \sigma^2$ , where  $\sigma_0^2$  is the variance at  $t = 0$ , which is reasonable to take as equal to 0. Therefore

$$\sigma^2 = \frac{\sigma_d^2}{\mu_d} \quad (25)$$

As a result of the above, the one step predicted distribution would be

$$P(x_t) = \alpha e^{-\frac{1}{2} \left( \frac{(x_t - (x_{t-1} + k))^2}{\sigma_{t-1}^2 + \sigma^2} \right)} \quad (26)$$

In the case presented above  $f(t)$  representing a best prediction of the way the order is expected to be evolving is a constant computed as the average speed between sequential checkpoints. If  $f(t)$  is not a constant, due for example to different traffic conditions during the day, the estimation of the required parameters becomes more complicated and approximations need to be made in order to get an estimate of their value based on historical observation data.

In the case where the location variable  $X_t$  is treated as discrete, one can use observation data to estimate the prior distribution, the transition model parameters and the sensor model probabilities using the Expectation-Maximization (EM) algorithm (Liao et al., 2003; Rabiner, 1989). The algorithm essentially finds the model parameter values that best fit the observed locations at each time step, therefore “training” the model to better reflect the observed data. Adopting a discrete location space is very often more convenient with regard to computational and mathematical complexity in order to populate the dynamic model parameters and execute the model.

### Example

We demonstrate the use of the model through a simple example. Let us assume we have a supply chain where one manufacturer sends orders to three distribution centers which distribute these orders to five warehouses as depicted in Figure 3. In our example we will follow the path shown with dotted lines.

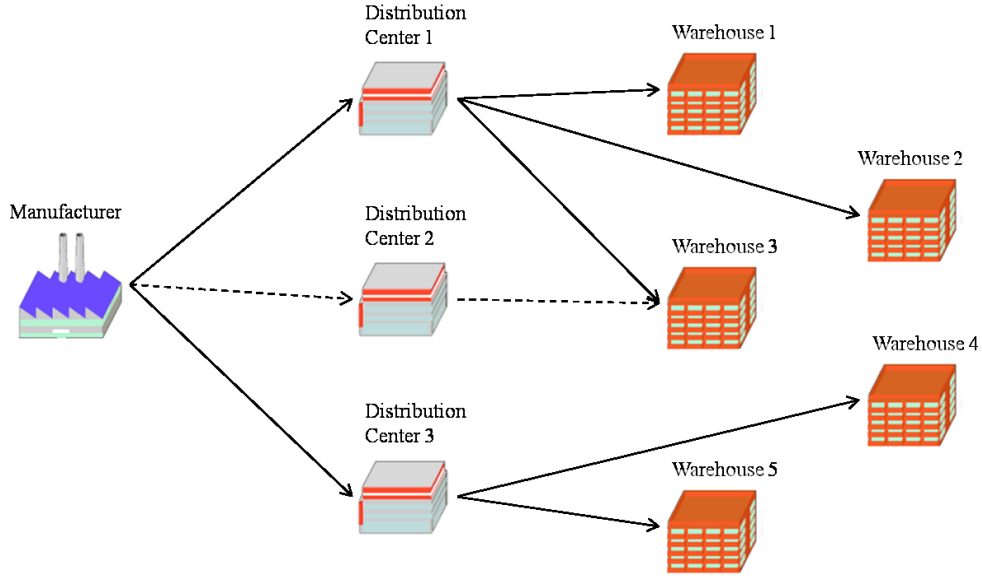


Figure 3: Supply chain example

Let us now assume that for this path, one checkpoint is installed in the "Distribution Center 2". We want to define the model parameters that will describe order movement in the supply chain and, given observation times at the checkpoint, to calculate the posterior location probability distribution at any time. This posterior distribution can then be used to answer some managerial questions (See Section "Applicability: where and how the model can be used"), in order to make operational decisions.

### Step 1: Static Information Definition

Let the required static information for the supply chain of our example be:

- The supply chain path is the one from the "Manufacturer" to the "Distribution Center 2" to the "Warehouse 3"
- The location  $X$  shall be measured in km and the location space is  $S = [0, 60]$ .
- There is one checkpoint at  $c_1 = 30$  km.
- Ultimately, we are interested in the posterior over the set of states  $Z = \{z^1, z^2, z^3\}$ ,  $z^1 = [0, 20)$ ,  $z^2 = [20, 40)$ ,  $z^3 = [40, 60]$ .

A summary of the static information of our example is shown in Figure 4.

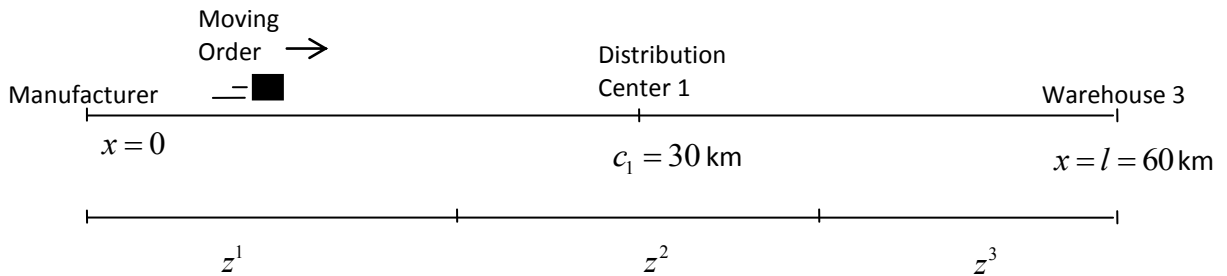


Figure 4: Example's static information

### Step 2: Historical Information Collection

We assume that we have collected historical observation data for 1000 shipments that were shipped from point  $x = 0$  km, passed and detected at  $c_1 = 30$  km and arrived at  $l = 60$  km. The data include observation

records at these three points. The mean time that orders needed to go from  $x = 0$  km to  $c_1 = 30$  km was  $\mu_d = 30$  mins with a variance of  $\sigma_d^2 = 21$  mins. For simplicity, we assume that the results were the same for the transport time between.

### Step 3: Model execution

Having collected the historical information, we can populate the dynamic model parameters and then calculate the posterior location distribution and posterior state distribution as described in subsection "How to calculate a posterior location probability distribution".

#### i. Populating the dynamic model parameters

- Prior distribution: All items start from location  $x = 0$  at  $t = 0$ . The prior distribution  $P(X_0)$  can be represented as a normal with  $\mu_0 = 0$  and a very small variance  $\sigma_0^2 \rightarrow 0$ .
- Transition model: based on what we described in subsection "Populating the model parameters", and using the historical data collected in the previous step we can populate the transition model parameters  $f(t)$  and  $\sigma^2$ . Assuming a constant  $f(t)$ , from (24) and (25) we have  $f(t) = 1 \text{ km/min}$  and  $\sigma^2 = 0.7 \text{ km}^2/\text{min}^2$ .
- Sensor model: taking into account the configuration of our tracking system, we select a sensor model that is equivalent to the one defined by (16), (17) and (19) in Table 2 and is given by:

$$\begin{aligned} P(Y^{c_i} = (0,0) | x_k) &= 1 \text{ when } x_k < c_1 \\ \text{and} & \\ P(Y^{c_i} = (0,0) | x_k) &= 0 \text{ when } x_k \geq c_1 \end{aligned} \quad (27)$$

$$\begin{aligned} P(Y^{c_i} = (1,1) | x_k) &= 1 \text{ when } x_k = c_1 \\ \text{and} & \\ P(Y^{c_i} = (1,1) | x_k) &= 0 \text{ when } x_k \neq c_1 \end{aligned} \quad (28)$$

$$\begin{aligned} P(Y^{c_i} = (0,1) | x_k) &= 1 \text{ when } x_k \geq c_1 \\ \text{and} & \\ P(Y^{c_i} = (0,1) | x_k) &= 0 \text{ when } x_k < c_1 \end{aligned} \quad (29)$$

#### ii. Posterior location probability distribution calculation

Starting from the prior distribution  $P(X_0)$  at  $t = 0$ , for each time  $t$  we apply the one step prediction (2) and then, based on the value of information signal  $Y_t^{c_i}$ , we calculate the revised posterior location probability distribution according to (7). This will result in a posterior location probability distribution for each time step. Figure 5 shows the posterior location probability distribution over time for an order that was detected at checkpoint  $c_1$  at time  $t = 40$  and Figure 6 shows the same distribution assuming that an order was detected at  $c_1$  at time  $t = 20$ . A slice across the time axis in the figures below gives the location probability distribution at that time  $t$ . This distribution is the answer to the question "what is/was the order's location at time  $t$ ?"

Figure 5 shows how the probability distribution is concentrated just before the checkpoint location as time passes by and the order has still not been observed there, whereas Figure 6 demonstrates how the distribution is revised once the observation becomes available at  $t = 20$ , even if the estimate just before that was much earlier than the checkpoint's location.

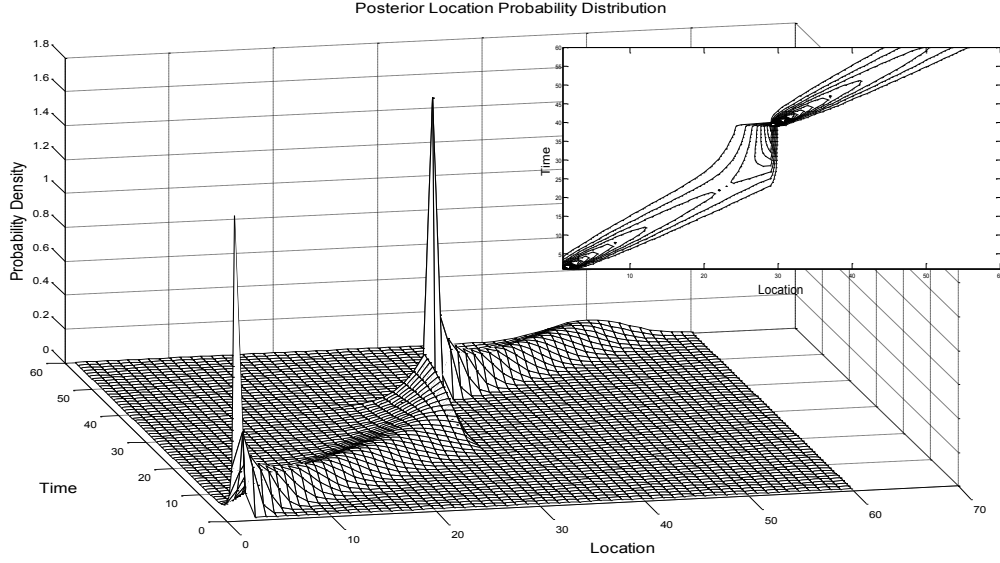


Figure 5: Posterior location probability distribution and contour plot. Order observed at checkpoint  $c_1$  at  $t = 40$ . Sensor model defined by (16), (17) and (19).

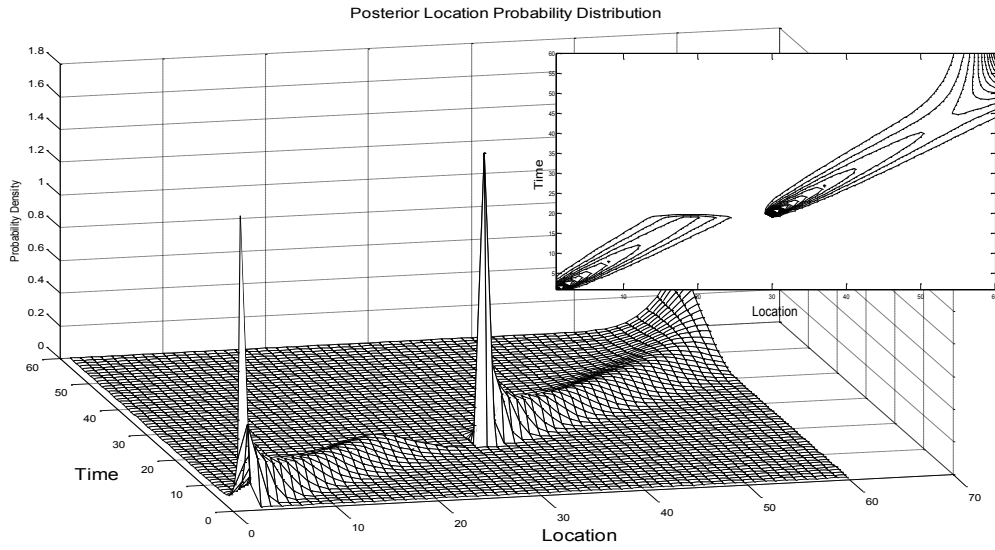


Figure 6: Posterior location probability distribution and contour plot. Order observed at checkpoint  $c_1$  at  $t = 20$ . Sensor model defined by (16), (17) and (19).

In order to demonstrate the role of the sensor model, Figure 7 shows the posterior distribution for the same detection time  $t = 40$  as in Figure 5 but using a sensor model described by (15), (17) and (18). The difference is that in the case of Figure 7 for  $t < 40$  the distribution spreads even after the checkpoint location  $c_1 = 30$  as defined by equation (15) of the sensor model, whereas in the case of Figure 5 the distribution is limited for locations  $x < c_1$  when  $t < 40$ , as defined by equation (16).

The ongoing location estimate shown in Figures 5-7 shows how the proposed model delivers enhanced tracking information over time, bridging the gaps between Auto-ID observations and taking into account expected order behavior and checkpoint configuration. The resulting tracking information



can be used in operational decision-making, enabling the evaluation of the information quality with regard to its ability to support these decisions.

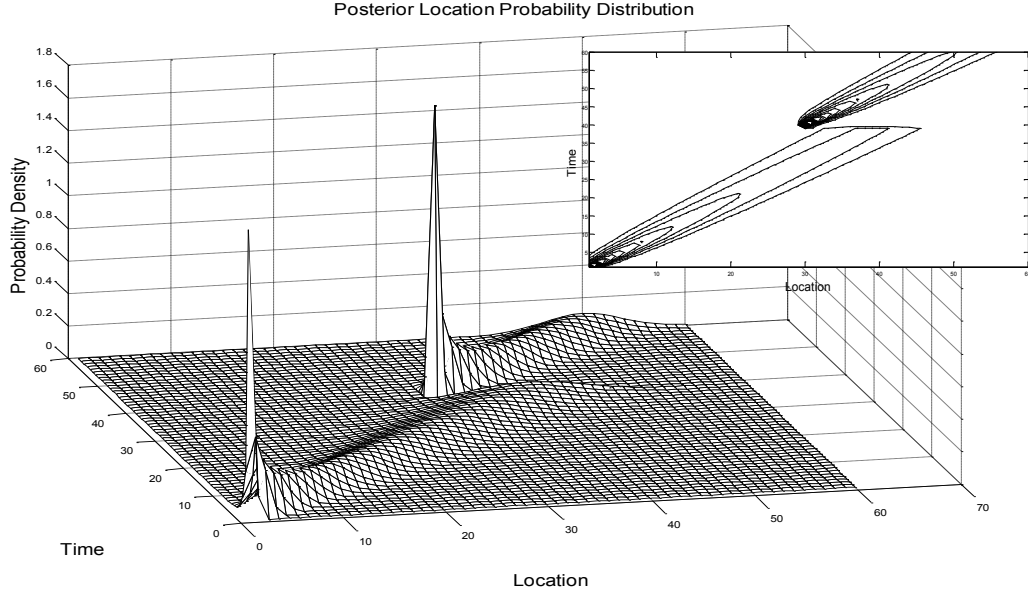


Figure 7: Posterior location probability distribution and contour plot. Order observed at checkpoint  $c_1$  at  $t = 40$ . Sensor model defined by (15), (17) and (18).

### iii. Posterior over $Z$

Once having calculated the posterior distribution, we use (10) to calculate the posterior over the set of states  $Z = \{z^1, z^2, z^3\}$  as defined in subsection "Static information definition". Figure 8 shows the posterior distribution over the states as a function of time.

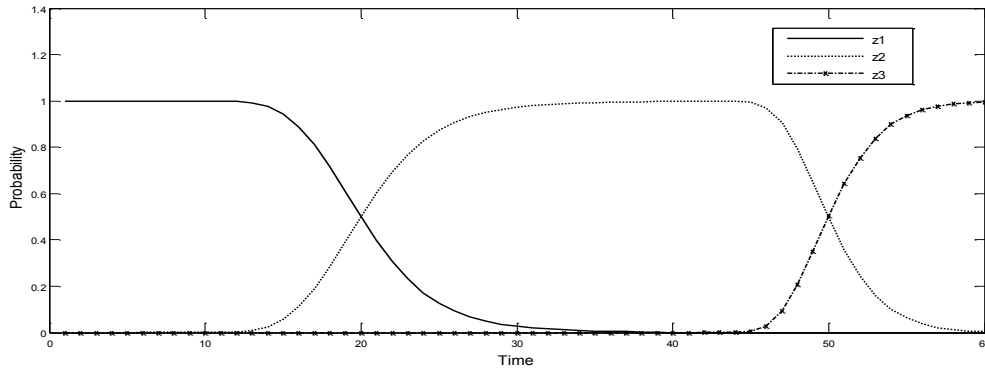


Figure 8: Posterior distribution over states  $Z$ . Order observed at checkpoint  $c_1$  at  $t = 40$ . Sensor model defined by (16), (17) and (19).

### Applicability: Where and How the Model can be Used

The proposed model can be used to help address operational problems that involve supply chain tracking information. The posterior location probability distribution generated as output from our model can be used as input into studies that use tracking information as one of their parameters. Decision effectiveness in inventory management, production scheduling or distribution planning based on supply chain tracking information are

some examples of research areas to which the proposed model is directly applicable. The model would allow researchers to study the impact of different parameters of the tracking system, such as checkpoint configuration or information accuracy, on the performance of the studied operations. Our research is currently extending the proposed model, in order to quantify the value of tracking information in supply chain decisions.

From an industrial perspective, the proposed model can be used in “intelligent” tracking systems that could deliver order location estimations based on observation information. These location estimates can be used by managers to answer questions regarding the ongoing location of products through their supply chains and make the necessary decisions. Examples of such questions are "What was the order's location an hour ago?" and "What is the probability that the order will be here by 2pm tomorrow". Moreover, the location estimates from these enhanced tracking systems can be used in higher level decision support systems, which could in turn use this information for optimizing their output. Finally, although the model is intended to focus on orders moving in the supply chain, the results are broader covering tracking information issues in other contexts too.

## CONCLUSIONS AND FURTHER WORK

This paper has proposed a mathematical model for order location estimation in a supply chain, based on expected order behavior (defined by the transition model) and observation information across the supply chain (defined by the observation signals and the sensor model). Given a set of order observation signals, the model generates a location probability distribution, which expresses the estimate at any time for the order's current or future location. We have described the way that the model can be applied in a supply chain tracking application and we have demonstrated its use through an example.

The proposed model assumes the existence of a probability distribution given a priori. Although different orders can in reality move in a different way in a supply chain, we believe that our approach applies to a big subset of situations where prior data for travel on segments of the supply chain is relevant. Our model presented here assumes a linear supply chain. Further work needs to be done to extend this for the case of a supply network, the first steps of which are presented in Kelepouris et al. (2011). The discrete version of the proposed model, presented in Kelepouris (2008), provides good potential for this purpose. Moreover, in the normal version of the model, we have assumed some properties of the transition model parameters which have made the calculations easier and clearer to present. In a real case, the parameters might not have these convenient properties and further research is needed to define a way in which the user of the model can estimate these values. Finally, an extension of the model needs to be studied for the case of noisy tracking signals (e.g. missed reads at checkpoints) and the case that multiple signals are used to produce a location estimate. Details of these extensions can be found in Kelepouris (2008).

We believe that this study provides a solid basis for studies on accurately assessing the potential of Auto-ID technologies for improving business operations through enhanced information availability. The location probability distribution that the model delivers can be used for studying the impact of tracking information on many higher-level industrial operations and quantifying its value for the decision-maker. We intend to present the results of our research in this direction in a later paper.

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